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Calculating the dissipation in fluid dampers with non-newtonian fluid models

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ABSTRACT

The present paper gives a comparison of the Maxwell, Upperconvected Maxwell and the Oldroyd-B model for the calculation of dissipation in high shear-rate cases. Usage of viscodampers in the automotive industry is the most common. There is a good scope of the computing this power in the case of Newtonian fluids. When a polymeric liquid is considered that part of energy that is irreversible cannot be calculated as $P_{diss.} = \boldsymbol{\tau} : \mathbf{d}$. For fluids where the separation into a solvent and a polymer part is not available but the deformation gradient tensor must be separated into two parts. One part consists of only the elastic deformation while the other is the non-elastic. This paper shows this separation using the Maxwell and the UCM models. A simple problem is shown, solving both analytically and numerically. The steady state temperature distribution of a damper then is validated with measurement.

1. Introduction

Many complex fluids of interest exhibit a combination of viscous and elastic behavior under strain. Examples of such fluids are polymer solutions and melts, oil, toothpaste, and clay, among many others. The Oldroyd-B fluid presents one of the simplest constitutive models capable of describing the viscoelastic behavior of dilute polymeric solutions under general flow conditions. Despite the apparent simplicity of the constitutive relation, the dynamics that arise in many flows are complicated enough to present a considerable challenge to numerical simulations. The flow of a polymeric or viscoelastic liquid is considered in this paper. These fluids have a very complex set of properties, the first and second normal stress coefficients ψ_1, ψ_2 can be observed. Unlike Newtonian fluids there are normal stresses which are unequal.

There are several fluids that cannot be described in such a simple way. These typically non-Newtonian fluids have much more complex features. These fluids include polymeric liquids, polymer melts, soap solutions, suspensions, and emulsions. The material parameters for Newtonian fluids depend on temperature and pressure but not on the velocity gradient. For non-Newtonian fluids these parameters highly depend on the velocity gradients and they produce elastic effects. In some circumstances these polymeric and other high viscosity fluids can produce heat. This production can be useful but sometimes needs to be resolved. In the field of engineering this is an important problem. For Newtonian fluids the dissipation can be calculated as

$$P_{diss.} = \boldsymbol{\tau} : \mathbf{d}, \tag{1}$$

where $\mathbf{d} = \frac{1}{2}\left(\nabla\mathbf{v} + \nabla\mathbf{v}^T\right)$. The calculation is well known [1–10]. This method is quite simple for Newtonian fluids and for the Maxwell model. For polymeric liquids, whose stress cannot be separated into a solvent and a liquid part, this computation leads to high errors if the dissipation is calculated as given in (1). The Maxwell model is widely used by engineers because of its simplicity. On the other hand this cannot be used as a constitutive equation because it is not objective. Viscoelastic models are also known for describing non-Newtonian fluids. These are good when we want to observe small displacements. The third way is the use of non-linear viscoelastic models. These are partial differential equations and give a very accurate presentment for these fluids.

In this paper the calculation of the dissipated power is presented if the material model is viscoelastic. The presented following method is compared in an analytical model and this result is used to check with measurements.

2. Governing equations

1 Continuity equation

$$\nabla\mathbf{u} = 0, \tag{2}$$

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where ∇ denotes the divergence operator.

2 Equation of motion

$$\frac{\partial}{\partial t}\rho\mathbf{u} = -\nabla\rho\mathbf{u}\mathbf{u} - \nabla\boldsymbol{\pi} + \rho\mathbf{g}, \tag{3}$$

where π is total stress tensor,

3 Energy equation

$$\frac{\partial}{\partial t}\rho U = -\nabla\rho U\mathbf{u} - \nabla\mathbf{q} - \boldsymbol{\pi} : \nabla\mathbf{u}, \tag{4}$$

where ρ denotes density, U is the internal energy per mass unit, $\mathbf{q}$ is the heat flux and the last term is dissipated power.

The equation of an equaly linear and elastic fluid is the Maxwell model

$$\boldsymbol{\tau} + \lambda\dot{\boldsymbol{\tau}} = -2\mu_0\mathbf{d}, \tag{5}$$

where $\lambda = \frac{\mu_0}{G}$ is the time constant (relaxation time) and μ_0 is the zero shear rate viscosity. This model is not suitable to use as a constitutive equation so the Upper Convected Maxwell (UCM) and Oldroyd-B model are introduced

$$\boldsymbol{\tau} + \lambda\overset{\triangledown}{\boldsymbol{\tau}} = -2\mu_0\mathbf{d}, \tag{6}$$

$$\boldsymbol{\tau} + \frac{\mu_0}{G}\left(\frac{\partial\boldsymbol{\tau}}{\partial t} + \mathbf{u}\cdot\nabla\boldsymbol{\tau} - (\nabla\mathbf{u})^T\boldsymbol{\tau} - \boldsymbol{\tau}(\nabla\mathbf{u})\right) = -2\mu_0\mathbf{d}. \tag{7}$$

The equation of the Oldroy-B fluid is

$$\boldsymbol{\tau} + \lambda\overset{\triangledown}{\boldsymbol{\tau}} = -2\mu_0(\mathbf{d} + \lambda_1\overset{\triangledown}{\mathbf{d}}), \tag{8}$$

where λ_1 is the retardation time.The problem introduced in this paper used cylindrical coordinate system. Due to the axial symmetry the following simplification can be taken

$$\frac{\partial}{\partial\varphi} \equiv 0. \tag{9}$$

3. Computing the dissipated power

In case of Newtonian fluids it is known that the dissipated power of such fluid can be calculated as Eq. (1). If the material model that is used to describe the behaviour of the fluid can be modelled as springs and dashpots connected in series and/or parallel Eq. (1) cannot be used. This way of calculating the dissipation gives higher values than real world measurement, because this takes into account the deformation of the spring(s). Instead of $\mathbf{d}$ a term that contains only the deformation(s) of the dashpots is needed. For this $\mathbf{d}$ is split into two parts. The equation of energy can be derived if the dot product of the equation of motion

$$\rho C_p\frac{DT}{Dt} = -(\nabla\mathbf{q}) - \left(\frac{\partial\ln\rho}{\partial\ln T}\right)_p\frac{Dp}{Dt} - (\boldsymbol{\tau} : \nabla\mathbf{u}). \tag{10}$$

The last term of the right side is the viscous dissipation. This part of the mechanical energy is irreversible. In most cases this term is negligible, but for high deformation gradients it may have a very significant value

$$P_{diss.} = -\boldsymbol{\tau} : \nabla\mathbf{u}. \tag{11}$$

The stress and the velocity gradient appear in the formulae. For each arbitrary material model the solution is to calculate $\boldsymbol{\tau}$ and insert it to the equation. For Newtonian fluids the viscous dissipation is always positive. The material law of such a fluid is rather simple, so it can be easily inserted into Eq. (11). Then it becomes

$$-\boldsymbol{\tau} : \nabla\mathbf{u} = \frac{1}{2}\mu_0\sum_i\sum_j\left[\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) - \frac{2}{3}(\nabla\mathbf{u})\delta_{ij}\right]^2 + \kappa(\nabla\mathbf{u})^2, \tag{12}$$

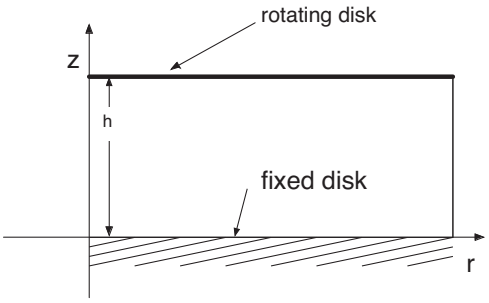

Fig. 1. The computational domain

where δ_{ij} is the Kronecker delta, which is 0 if $i \neq j$ and 1 if $i = j$ and κ is the diffusivity of fluid.

For those polymers that can be separated to a solvent and a polymer section, such that $\mu = \mu_s + \mu_p$ holds, the dissipation can be calculated as Eq. (11). The original calculation leads to high errors if the dissipated power is needed.

The Maxwell model cannot be used in this way. The material law of such fluid cannot be arrenged for the stress $\boldsymbol{\tau}$ in a closed form so it cannot be inserted into Eq. (11). So the other way is to try to calculate the anoter term in Eq. (11) to calculate the plastic deformation. Different material models can be built by springs and dashpots connected parallel and/or in series. If this model contains dashpots and springs the calculation of the viscous dissipation needs only those deformations that occur in the dashpots. The deformation of the springs are reversible. So the rate of strain tensor should be separated. Suppose that the deformation gradient tensor $\mathbf{d}$ can be written as

$$\mathbf{d} = \underset{d}{\dot{\boldsymbol{\gamma}}} + \underset{s}{\dot{\boldsymbol{\gamma}}}, \tag{13}$$

where $\dot{\boldsymbol{\gamma}}_d$ is the rate of strain tensor of the dashpot and $\dot{\boldsymbol{\gamma}}_s$ is the rate of strain of the springs.

Namely the rate of strain tensor can be separated into the sum of two tensors. One describes the deformation of the elastic parts of the model while the other the plastic deformation. By definition, analogous to Hooke's law in strength of materials the reversible part is

$$\underset{s}{\dot{\boldsymbol{\gamma}}} := \frac{1}{G}\frac{\partial\boldsymbol{\tau}}{\partial t}, \tag{14}$$

$$\frac{\mathcal{D}}{\mathcal{D}t}\boldsymbol{\alpha} = \frac{D}{Dt}\boldsymbol{\alpha} + \frac{1}{2}\left(\boldsymbol{\beta}\cdot\boldsymbol{\alpha} - \boldsymbol{\alpha}\cdot\boldsymbol{\beta}\right), \tag{15}$$

where $\boldsymbol{\beta} = \nabla\mathbf{u} - (\nabla\mathbf{u})^T$ and $\boldsymbol{\alpha}$ is arbitrary tensor.

Thus for a fluid with arbitrary material constitutive equation the viscous dissipation can be calculated as

$$P_{diss.} = \boldsymbol{\tau} : \frac{1}{2}\dot{\boldsymbol{\gamma}}_d. \tag{16}$$

Here

$$\dot{\boldsymbol{\gamma}}_d = \frac{1}{2}\left(\nabla\mathbf{v} + \nabla\mathbf{v}^T\right) - \frac{1}{G}\left(\frac{\partial\boldsymbol{\tau}}{\partial t} + \mathbf{v}\nabla\boldsymbol{\tau}\right). \tag{17}$$

3.1. Analytical solution in cylindrical coordinate system

For comparing the two chosen material models, the Maxwell and the UCM, axisymmetric geometry was used. The computational domain can be seen in Fig. 1. The velocity vector is

$$\mathbf{u}_{r,\phi,z} = \begin{bmatrix} 0 \\ \frac{rz}{h}\Omega_0\sin\omega t \\ 0 \end{bmatrix},$$