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On the plastic zone size of solids containing doubly periodic rectangular-shaped arrays of cracks under longitudinal shear

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ABSTRACT

Multiple cracks interaction plays an important role in fracture behavior of materials. A number of studies have been devoted to analytical and numerical analyses of the doubly periodic arrays of cracks. A very natural and highly accurate solution procedure is proposed to describe the interaction effect among the doubly periodic rectangular-shaped arrays of cracks. The proposed solution is implemented in the framework of continuously distributed dislocation model and singular integral equation approach. The accuracy of this solution is proved through a comparison of results from the present simulation and known closed form solutions. Further, the interaction effects among the periodic cracks on the plastic zone size and crack tip opening displacement are studied. It is found that the interaction distance among the vertical and horizontal periodic cracks is quite different.

1. Introduction

In this paper, we shall discuss a problem in the theory of elastic-plastic solids during a study of doubly periodic rectangular-shaped arrays of cracks in an infinite elastic-plastic solid. The exact determination of the influence of plastic yielding on the stress and deformation near the root of the cracks or notches is of basic importance for such elastic-plastic fracture mechanics problems [1]. Historically the Irwin model [2] and Dugdale model [3] are two such methods used to estimate the plastic zone size (PZS) for a crack in homogeneous materials. Dugdale [3] proposed one method for removing this singularity near the crack tip from the angle of linear-elastic fracture mechanics. The requirement is made that near the edge of the crack a plastic yield condition is satisfied. The region over which yield occurs is determined by requiring that the stresses remain finite everywhere. An alternate approach is the simpler elastic-plastic problems under longitudinal shear [4]. Afterwards, Barenblatt [5] presented another method to cope with this problem. He postulated a system of stresses existing at the edge of the crack that inhibits the occurrence of infinite stresses at the edge of the crack. These he called “cohesive stresses” and he developed a universal constant called the Modulus of cohesive to define the stresses. The discussions about the single crack and the infinite sequence of cracks from the standpoint of the theory of dislocations of dislocations have also been completed [6–8]. Two isolated crack problems containing a slit under anti-plane shear and a penny-shaped crack are studied by Keer and Mura [9]. In their work, a Tresca yield condition is used in both cases to ensure that all stresses are bounded. An exact linear elastic-perfectly plastic solution is presented for the problem of a sharp notch (or, when the notch angle is zero, a crack) in a plane of finite width subjected to anti-plane stresses inducing a stress and deformation stare of longitudinal shear by Rice [1]. The behavior of a crack under longitudinal shear taking some account of plasticity and cohesive forces near the crack tips is considered by Kostrov and Nikitin [10]. Panasyuk and Savruk [11] surveyed theoretical solutions to elastoplastic treatments in which the plastic zones are simulated as plasticity bands. Using the solution of an edge dislocation interacting with a circular inclusion as the Green's function, recently, Hoh et al. [12] investigated the Dugdale crack behavior near a circular inclusion.

The fracture problem of a doubly periodic array of cracks contained in an infinite solid has been a continuing problem for decades. There are in essence two methods to the solution of the problem. The first class of approaches is based on Muskhelishvili's complex potential method. Firstly, there is the boundary collocation approach of Isida and Igawa [13] using a combination of Muskhelishvili's complex potential and distributed force doublets.

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In addition, based on the complex potentials of Muskhelishvili, Ioakimidis and Theocaris [14] dealt with curved cracks in doubly periodic arrays by using the doubly periodic functions. By combining the elliptical function theory and conformal mapping technique, Hao [15] obtained an exact solution to the antiplane problem of doubly periodic cracks for the first time. Tong et al. [16] improved Hao's work sufficiently, and derived a closed form solution for an isotropic piezoelectric material with a doubly periodic array of cracks under far-field antiplane mechanical load and inplane electrical field. Under the similar method, the further studies of doubly periodic array of cracks or rigid-line in different materials are discussed completely [17–19]. Besides, Li [20,21] used complex potential methods to solve the fundamental complete plane strain problems of a three dimensional nonhomogeneous elastic body with a doubly periodic set of cracks or cylindrical inlay. The second method to solve the doubly periodic array of cracking problem relies on the equivalence between cracks and appropriate dislocations (i.e. the continuously distributed dislocation model). Bilby and Eshelby [22] first proposed the method of distributed dislocations to solve the crack problem. Karihaloo [23] extended his work, modeling the stress relaxation process at the tip of each crack in rectangular and diamond shaped arrays. But some errors often occur due to the convergence obstacle of double infinite summation in the solution procedure. Finally, the numerical accuracy of the solution procedure has been well solved by Karihaloo et al. [24] and Karihaloo and Wang [25]. In addition, researches of effective elastic properties of cracked solids have also been widely concerned. Nemat-Nasser et al. [26] and Nemat-Nasser and Yu [27] presented a systematic method for estimating the overall properties of solids with periodically distributed cracks. In their work, the overall elastic moduli, crack opening displacements and stress intensity factors for a more general and complex doubly periodic crack model (i.e. solids with periodically distributed perpendicular 2-D line cracks) are studied. Besides, Kachanov and his co-authors presented a series of studies to explicate the effect of crack interactions on overall mechanical behavior and emphasized particularly that crack interaction has a significant influence on macroscopic mechanical properties. Kachanov [28] pointed out that the crack interaction problems can be formulated in terms of the interaction tractions. His simple model sheds some light on the character of interactions and clarifies the nature of various approximation processes. Kachanov's method is applied to study the intersecting cracks problem, in both 2-D [29] and 3-D [30] configurations. In the problem of the effective elastic properties of cracked solids, this simple method is also sufficient for the numerical simulations of the effective properties [31].

The present work constitutes a continuing study in questing for a very natural and highly accurate solution procedure to study the elastic-plastic behavior for the doubly periodic rectangular-shaped arrays of cracks under the longitudinal shear. If a body contains a number of internal cracks, the spread of plasticity from any one is influenced by the presence of the others and a change in behavior is again expected when the plasticity has spread completely from one to another. However, it is seen that elastic-plastic solutions to the problems of doubly periodic arrays of cracks are very limited, and are highly desirable and attractive. In the framework of continuously distributed dislocation model, a new method takes full advantage of the periodic symmetry to avoid the appearance of double infinite summation, which is wrongly considered inevitable using such dislocation model. Thus, errors of a double infinite summation are disappeared, and the accuracy increases. The present solution procedure can be made as accurate as desired from the closed form solution for the linear elastic fracture problem of doubly periodic rectangular-shaped arrays of cracks. To illustrate this point, the results are also compared with those of Tong et al. [16] who obtained a closed form solution for a doubly periodic array of cracks. The focus is put on the revelation of the fracture characteristics and the prediction of the effective property of such cracked materials. To this end, the interaction effect among the doubly periodic rectangular-shaped arrays of cracks on the Plastic zone size (PZS) and crack tip opening displacement (CTOD) is studied.

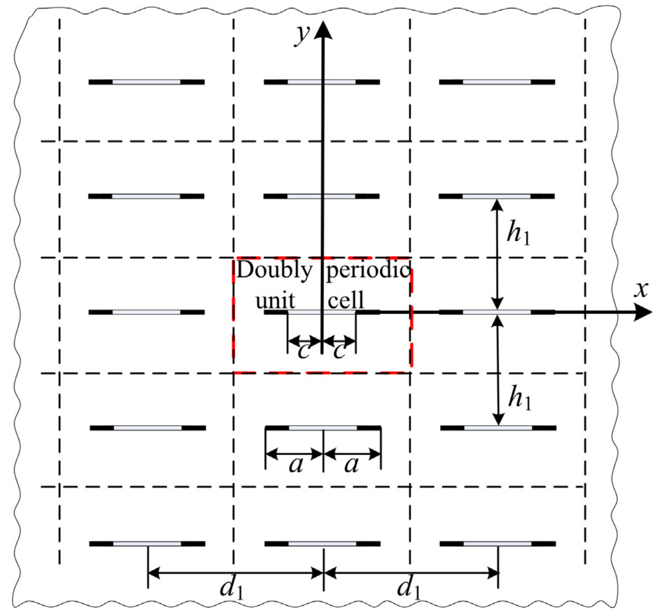

Fig. 1. Doubly periodic rectangular-shaped arrays of cracks.

2. Formulation of the problem

Antiplane deformation or longitudinal shear occurs in the deformation of a body with constant cross section by forces parallel to the axis (z axis) and identical for all cross sections. We consider here cases where all components of stress and strain depend on two Cartesian coordinates x and y. Further, attention is restricted to homogeneous, and isotropic elastoplastic materials. We consider longitudinal shear by forces $\tau_{zy}=\tau_0$ at infinity for an ideally elastic solid containing a doubly periodic rectangular-shaped array of cracks is shown in Fg.1. The local coordinates of the composite are also displayed in Fig. 1. The z-axis is perpendicular to the x-y plane which is the isotropic plane of the materials. The rectangular coordinate systems are established with the origin located at the center of the crack. The traction-free cracks occupy the positions md_1-$c \le x \le md_1+c$, $y \le nh_1$,$(m,n=0,\pm1,\pm2,\ldots)$, while the plasticity bands are located on the continuation of the crack md_1-$a \le x \le md_1$-c and $md_1+c \le x \le md_1+a$, $y \le nh_1$. As presented in Fig. 1; each crack in the rectangular array is of length $2c$, and is separated from other cracks by a distance h_1 vertically and a distance d_1 horizontally. h_1 and d_1 are the vertical and horizontal periodic parameters, respectively. Studies have been made on the plastic strain in a body containing longitudinal shearing cracks, which showed that the plastic strain is not localized in thin layers in the case of an ideally elastoplastic material or a hardening one. Nevertheless, the model for a crack with plasticity bands is widely used in the failure mechanics of antiplane deformation. The tangential stresses $\tau_{zy}=\tau_Y$ are given on the plasticity bands, while behind them the material is elastic.

The doubly periodic rectangular-shaped arrays of cracks problem is periodic with respect to the x-axis, as well as with respect to the y-axis. Therefore, our attention is confined to the doubly periodic unit cell defined by $-d_1/2 \le x \le d_1/2$, $-h_1/2 \le y \le h_1/2$ as shown in Fig. 2. Hereafter, the subscripts/superscripts 1 and 2 denote the