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A new formulation for solving 3-D time dependent rolling contact problems of a rigid body on a viscoelastic half-space

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ABSTRACT

The paper deals with a new formulation for solving the rolling contact problem without friction of a rigid body on a viscoelastic half-space in three dimensions. Assuming that the material behavior is independent of time for a sufficiently short time duration, the viscoelastic contact problem is transformed into elastic like problems. Then the contact problem is solved using a direct numerical method at each time step. The convergence of the method in time and space is good for a spherical indenter. The dissymmetry of the contact patch due to hysteresis was found in three dimensions for the spherical indenter and two cylinders of different width. Finally the method was tested for a sinusoidal varying speed and shows a good efficiency.

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1. Introduction

Rolling systems such as car tyres and conveyer systems widely use elastomers. Therefore the time dependent behavior of elastomers should be taken into account when computing rolling contact for such systems. The viscoelastic rolling contact is also a fundamental problem which was first investigated by the experimental work of Tabor [17] and the studies of Hunter [7] and Morland [13] on the rolling contact of a rigid cylinder on a viscoelastic half-space. However most of these studies are restricted to cylinder cases for two dimensional problems. This paper presents a new algorithm for computing the 3-D time dependent rolling contact between a rigid body and a viscoelastic half-space. The friction is not taken into account in this work.

Since the middle of the 1980s, Finite Element Methods (FEM) have been used to solve viscoelastic rolling contact problems. Padovan and Paramadilok [16] developed a travelling finite element strategy based on moving total Lagrangian coordinates to handle transient and steady viscoelastic rolling contact. Oden and Lin [15] studied the contact of hyperelastic and viscoelastic rolling cylinders on a rough foundation. The Arbitrary Lagrangian Eulerian (ALE) formulation and a finite element approach were applied to rolling contact problems by Nackenhorst [14] and later applied to rolling noise simulations [1].

On the other hand, Boundary Element Methods (BEM) were used by Kalker to solve the elastic rolling contact problems [9] and later the problem of rolling viscoelastic multilayered cylinders [10]. The contact stresses between viscoelastic cylinders were also computed with BEM by Wang [18] and González and Abascal [6].

Though many formulations and strategies have been developed to solve viscoelastic rolling problems, most of them are restricted to 2-D and/or stationary rolling. In this paper, a new formulation is proposed for the 3-D time dependent rolling contact between a rigid body and a viscoelastic half-space. The friction will be neglected in the present work and only the normal pressure is studied. Based on the assumption that the material behavior is independent of time for a sufficiently short time duration, the viscoelastic contact problem is transformed into elastic like problems. As a result, numerical methods developed for solving elastic contact problems can be used.

In Section 2, the formulation of viscoelastic contact is described without the rolling conditions for facilitating the comprehension. Then the rolling conditions are introduced in a Lagrangian coordinates system. Section 3 presents the numerical algorithm that uses the principle of direct matrix inversion methods [8]. Numerical results are given in Section 4 for a rolling sphere and rolling cylinders of different lengths. The method and the examples are presented in terms of a displacement-control problem. Transient rolling and sinusoidal speed rolling are considered to show the efficiency of the formulation.

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2. Formulation of the viscoelastic rolling contact problem

The problem of the contact between a rigid solid and a viscoelastic half-space is considered in frictionless conditions. The present normal displacement w at any point (x, y) within the contact area depends on the pressure distribution history $p(\xi, \eta, \tau)$:

$$w(x,y,t)=\int_0^t J(t-\tau)\frac{d}{d\tau}\left[\int\int_{S_m} T(x,y;\xi,\eta)p(\xi,\eta,\tau)ds\right]d\tau \quad (1)$$

where S_m is the maximal contact area, J is the creep function and $T(x, y; \xi, \eta)$ is the influence function which designates the displacement induced at point (x, y) by a unit point force at (ξ, η).

The influence function is then given by the Boussinesq's fundamental solution:

$$T(x,y;\xi,\eta)=\frac{(1-\nu)}{\pi\sqrt{(x-\xi)^2+(y-\eta)^2}} \quad (2)$$

We suppose that the pressure distribution up to $t-\Delta t$ is known, and propose to compute the pressure distribution at instant t. For the sake of simplicity, we use the following notation for any two instants t_1 and t_2:

$$I_{t_1}^{t_2}=\int_{t_1}^{t_2} J(t-\tau)\frac{d}{d\tau}\left[\int\int_{S_m} T(x,y;\xi,\eta)p(\xi,\eta,\tau)ds\right]d\tau \quad (3)$$

Then the following equation holds:

$$I_0^t=I_0^{t-\Delta t}+I_{t-\Delta t}^t \quad (4)$$

In the latter equation, the first term $I_0^{t-\Delta t}$ can be computed from the pressure distribution history before $t-\Delta t$. The second term $I_{t-\Delta t}^t$ will be studied. When τ varies from $t-\Delta t$ to t, $t-\tau$ varies from Δt to 0. We assume that if the time duration Δt is sufficiently short, the creep function is constant:

$$J(t-\tau)\approx J(0) \quad \text{for } t-\Delta t<\tau<t \quad (5)$$

Then one can derive:

$$I_{t-\Delta t}^t\approx J(0)\int_{S(t)} T(x,y;\xi,\eta)p(\xi,\eta,t)ds \\ -J(0)\int_{S_m} T(x,y;\xi,\eta)p(\xi,\eta,t-\Delta t)ds \quad (6)$$

where $S(t)$ denotes the present contact area.

By introducing the preceding displacement $u(x, y, t)$ which represents the contribution to $w(x, y, t)$ due to the pressure distribution history between 0 and $t-\Delta t$:

$$u(x,y,t)=I_0^{t-\Delta t}-J(0)\int_{S_m} T(x,y;\xi,\eta)p(\xi,\eta,t-\Delta t)ds \quad (7)$$

and taking into account the above equations, Eq. (1) becomes:

$$w(x,y,t)\approx u(x,y,t)+J(0)\int_{S(t)} T(x,y;\xi,\eta)p(\xi,\eta,t)ds \quad (8)$$

Then the present pressure distribution can be determined by solving the unilateral contact problem given by the complementary relations between the gap and the normal pressure:

$$\begin{cases} \forall(x,y)\in S(t), & \delta(t)-z(x,y)-w(x,y,t)=0 \quad \text{and} \quad p(x,y,t)>0 \\ \forall(x,y)\in \bar{S}(t), & \delta(t)-z(x,y)-w(x,y,t)>0 \quad \text{and} \quad p(x,y,t)=0 \end{cases} \quad (9)$$

where $\delta(t)$ is the normal penetration of the indenter in the half-space (i.e. the normal displacement at the tip of the indenter), $z(x, y)$ describes the geometry of the surface of the indenter, $w(x, y, t)$ is the present normal displacement as defined in Eq. (1) and $\bar{S}(t)$ is the surface of the half-space where there is no contact. Then the contact condition on the gap in Eq. (9) combined with Eq. (8) leads to:

$$\forall(x,y)\in S(t),\quad J(0)\int_{S(t)} T(x,y;\xi,\eta)p(\xi,\eta,t)ds\approx\delta(t) \\ -z(x,y)-u(x,y,t) \quad (10)$$

Since $u(x, y, t)$ is assumed to be known, this problem can be seen as an elastic like contact problem at instant t.

In rolling conditions, one can follow a point in the contact plane by introducing the coordinates below:

$$X(x,y,t)=x+\int_0^t V_x(\tau)d\tau,\quad Y(x,y,t)=y+\int_0^t V_y(\tau)d\tau \quad (11)$$

where V_x and V_y designate the speed of the rolling solid in the contact plan. The frames of coordinates are illustrated in Fig. 1.

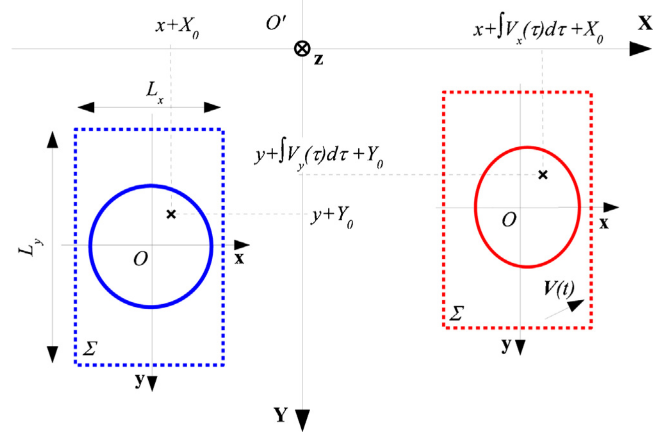

Fig. 1. Frames of coordinates for the rolling problem.

Then in rolling conditions one should replace in all equations (x, y) with (X, Y) and $z(x, y)$ with $Z(X, Y, t)$ which describes the surface profile of the rolling solid potentially in contact with the half-space at the present instant. For instance Eq. (1) becomes:

$$w(X,Y,t)=\int_0^t J(t-\tau)\frac{d}{d\tau}\left[\int\int_{S_m} T(X,Y;\xi,\eta)p(\xi,\eta,\tau)ds\right]d\tau \quad (12)$$

and the contact problem becomes:

$$\forall(X,Y)\in S(t),\quad \begin{cases} J(0)\int_{S(t)} T(X,Y;\xi,\eta)p(\xi,\eta,t)ds\approx\delta(t) \\ \qquad -Z(X,Y,t)-u(X,Y,t) \\ p(X,Y,t)>0 \end{cases} \quad (13)$$

Like Eq. (10), Eq. (13) is an elastic like problem which can be solved by using classical methods such as the Matrix Inversion Method [8,9].

3. Numerical procedure

The surface involving the potential contact area of size $L_x \times L_y$ was meshed using $n=n_x n_y$ rectangular elements of dimensions $dx=L_x/n_x$ and $dy=L_y/n_y$ centered at (x_i, y_i) and with uniform pressure on each element.

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