Mechanics Research Communications 64 (2015) 23–28

Contents lists available at ScienceDirect

Mechanics Research Communications

journal homepage: www.elsevier.com/locate/mechrescom

# A non-intrusive methodology for the representation of crack growth stochastic processes

Rafael Holdorf Lopez [a,*], Cláudio R. Ávila da Silva Jr. [b,c]

[a] Center for Optimization and Reliability in Engineering (CORE), Department of Civil Engineering, Universidade Federal de Santa Catarina, Florianópolis, Brazil
[b] NuMAT/PPGEM, Federal University of Technology of Parana, Curitiba, PR, Brazil
[c] PPGMNE/CESEC, Federal University of Parana, Curitiba, PR, Brazil

ARTICLE INFO

*Article history:*
Received 8 May 2014
Received in revised form
16 December 2014
Accepted 28 December 2014
Available online 6 January 2015

*Keywords:*
Fracture mechanics
Polynomial chaos
Stochastic process
Uncertainty quantification
Paris–Erdogan law

ABSTRACT

In this paper, we present a methodology to pursue the uncertainty quantification of the stochastic process that represents the crack growth problem. The main idea of this methodology is to discretize the crack growth process in a sequence of random variables and then, approximate each of them using a stochastic polynomial approach. This methodology is non-intrusive, i.e. it is based on the representation of random variables using stochastic polynomials, whose coefficients are evaluated using a least squares method and only a few realizations of the stochastic process. The Paris–Erdogan law was used as crack growth model in order to focus the reader's attention on the uncertainty quantification methodology. We modeled the parameters of the Paris–Erdogan law as random variables, i.e. the initial crack length and the coefficients of the Paris–Erdogan model are treated as random variables. Two numerical examples are presented in order to shown the effectiveness and accuracy of the proposed methodology. From the results of these examples, it is shown that the proposed methodology is able to successfully approximate the stochastic process that represents the crack growth for the Paris–Erdogan model, with a much lower computational cost than the MCS. The main limitation of the proposed approach is that, in the form it was presented, it is not able to handle random processes as input parameters.



## 1. Introduction

The analysis of the performance of structural components is more realistic when the existence of cracks is considered. The occurrence of cracks, when these are smaller than given thresholds, does not lead to the substitution of the structural component neither to its removal from operation. The concept of damage tolerant design, largely employed in the aircraft industry, may be cited here. Its object is to ensure that the structure will continue to sustain a high proportion of its design load even after damage has occurred [1]. In order to apply it, one must have an adequate system of inspection prescribed so that the damage may be detected and repaired. Consequently, the modeling of the damage (or crack growth) assumes an important role in the inspection and maintenance policies in structural engineering.

There is a consensus that the most appropriate model is one which takes into account the uncertainties inherent to the process of crack propagation. Among many works that consider and propose methods to quantify uncertainty, we may cite the works of Hoeppner and Krupp [2], Tanaka et al. (1981), Kozin and Bogdanoff [3], Lin and Yang [4], Castillo et al. [5], Fabro et al. [6] and Gomes and Beck [7]. The works of Ghonem and Dore [8] and Xiao et al. [9] classify the uncertainty quantification methods for the process of crack growth in two classes: statistic and stochastic. In the class of *statistical methods*, the idea is the randomization of some model of crack growth, which consists in modeling the parameters of the model as random variables. In *stochastic methods*, the parameters of the crack growth model vary, for instance, with time, i.e. the input parameters of the model are stochastic processes themselves. Nevertheless, all the proposed methods, in both approaches, must employ in their development a model or law for the crack growth. Such models, from the simplest to the most complex, have to make assumptions, which in most cases come from experimental observation, e.g. the Paris–Erdogan law [10]. Regardless of theoretical rigor of a given model, its success is determined by its use and performance in application to engineering problems, or even in its use as a foundation for new models.

* Corresponding author at: Universidade Federal de Santa Catarina, R. João Pio Duarte Silva, s/n, 88040-900 Florianópolis, Brazil. Tel.: +55 4837215175; fax: +55 4837219939.
*E-mail addresses:* rafael.holdorf@ufsc.br (R.H. Lopez), avila@utfpr.edu.br (C.R. Ávila da Silva Jr.).

http://dx.doi.org/10.1016/j.mechrescom.2014.12.005
0093-6413/

At the end of the eighties, Ghanem and Spanos [11] presented an innovative idea for the representation of stochastic processes in engineering problems, using chaos polynomials. This work presented to the engineering and applied mathematics communities a new approach for the uncertainty quantification in stochastic systems. It gave a new alternative for the representation of stochastic systems, besides the well-known methods based on perturbation and Monte Carlo simulation. This approach spread out in the form of new theoretical developments and applications in several fields. The existing polynomial chaos approaches may be classified in intrusive or non-intrusive, according to the fact that the original system is or not manipulated in order to introduce the expansion. In the non-intrusive approach, the expansion is not introduced in the model but used only on observed values. For example, the response of the system for different values of the input parameters is collected and analyzed a posterior in order to generate a representation.

In this context, this paper presents an uncertainty quantification methodology for the approximation of the stochastic process of crack growth. This methodology discretize the crack growth process in a sequence of random variables and employ the stochastic polynomial approach presented by Lopez et al. [12] to approximate each one of them. The main idea of the proposed approach is that from a small set of realizations of the stochastic process of the crack growth, the deterministic coefficients of the stochastic polynomials are computed using a linear least squares method, which is a non-intrusive approach. The proposed uncertainty quantification methodology is applied to the crack growth model of Paris–Erdogan. The uncertainty of the input parameters of this law is taken into account by a probabilistic parametric approach. That is, the proposed methodology is a statistical method, dealing with crack growth processes whose input parameters are random variables.

The main contributions of the paper, in the authors' point of view, are that (i) the proposed methodology is easy to implement, black box and require a low computational cost for the solution of the uncertainty quantification problem, (ii) it does not require any transformation of random variables when dealing with different probabilistic distributions in the same problem. It must be remarked here that the simple model given by the Paris–Erdogan law was employed in order to focus the reader's attention on the uncertainty quantification methodology and that it may be easily extended to more complex cases.

This paper is organized as follows: the proposed methodology is presented in Section 2, while Section 3 details its application to the Paris–Erdogan model. The numerical analysis of two examples is pursued in Section 4 and the main conclusions drawn from this work are summarized in Section 5.

## 2. The proposed methodology

In this section, we present the proposed methodology to pursue the approximation of the crack growth problem. The main idea of the proposed methodology is to discretize the crack growth process into a finite number of intervals, and in each one of them we approximate the size of the crack as a random variable.

In order to approximate each random variable of the discretization of the crack growth process, the proposed methodology utilizes stochastic polynomials. That is, the crack size for a given cycle number $N$ is determined by an expansion of $a = a(N, \xi(\omega))$ as a function of the random vector $\xi(\omega) = [\xi_1(\omega), \ldots, \xi_q(\omega)]^T$ on a Hilbertian basis $\mathcal{H}$ – such as, for instance, polynomial chaos expansions, [13,14], where "$q$" is the number of random variables employed in the expansion. In practice, a finite expansion with "$n$" terms is employed in order to obtain a finite-dimensional approximation $\mathcal{P}_a$ – the projection of $a = a(N, \xi(\omega))$ onto a convenient finite-dimensional space $\mathcal{S} \subset L^2(\Omega, F, P)$, i.e., $a \approx \mathcal{P}_a$, where:

$$\mathcal{S} = span\{\psi_i(\xi(\omega))\}_{i=1}^{n} = \left\{ a \in \mathcal{S} \,\middle|\, a = \sum_{i=1}^{n} a_i \psi_i(\xi(\omega)); \quad a_i \in \mathbb{R},\ 1 \le i \le n \right\}. \tag{1}$$

For a given basis $\Psi(\xi(\omega)) = [\psi_1(\xi(\omega)), \ldots, \psi_n(\xi(\omega))]^T$ of $\mathcal{S}$, we have that $\forall y \in \mathcal{S}$, $y_i \in \mathbb{R}$ such as $y = \sum_{i=1}^{n} y_i \psi_i$. Hence, $\mathcal{P}_a \in \mathcal{S}$ admits the following representation,

$$\mathcal{P}_a(N, \xi(\omega)) = \sum_{i=1}^{n} a_i(N) \psi_i(\xi(\omega)). \tag{2}$$

In order to construct this approximation, the random variables $\xi$ of the basis have to be chosen and the coefficients of the projection $\mathcal{P}_a$ have to be determined. Lopez et al. [12] presented a methodology that sets $\xi$ as the input random parameters of the problem. One of the advantages of this procedure is that it does not require any probabilistic transformation to handle different distributions since the expansion of Eq. (2) is done directly in terms of the input random parameters of the problem. Also, these authors tested several methods to compute $\mathcal{P}_a$, and from this comparison, the authors showed that the collocation approach provided the best results regarding computational cost and accuracy. The collocation approach generates a sample from the stochastic process that describes $a$, then the approximation is determined by using the known values as collocation points, i.e., $\mathcal{P}_a$ is determined as the best fit to these known values. The collocation approach is detailed in the following paragraphs.

Consider the set of realizations of the crack growth process $\{a(N_i; \xi_j)\}_{i,j=1}^{n_{cycles},\, n_s}$, obtained using a MCS, where $\xi_j$ is a realization of the random vector comprised by the random parameters of the crack growth model, and "$n_s$" is the number of samples/realizations of the MCS. In general, these realizations are obtained by the numerical approximation of the solution of the Initial Value Problem (IVP) chosen to describe the crack growth model for each realization of its random parameters. For example, in the next section, the IVP is given by the Paris–Erdogan model and the random vector is comprised by the parameters $m$ and $C$ of this model as well as the initial crack size.

As already mentioned, the stochastic process is discretized in a sequence of random variables, each defined by $N = N_i$. Thus, consider the set $\mathcal{B}_i = \{a(N_i; \xi_1), \ldots, a(N_i; \xi_{n_s})\}$ of the realization of each random variable generated by fixing $N = N_i$. From the set $\mathcal{B}_i$, we formulate an optimization problem, whose solution are the coefficients of Eq. (2). To accomplish that, a method of linear least squares is employed,

$$\begin{cases} \text{Find} \quad \boldsymbol{a}_i^* \in \mathbb{R}^n, \text{ such that,} \\ \boldsymbol{a}_i^* = \underset{a(N_i;\xi)\in\mathcal{B}_i}{arg\ \min}\{(\tfrac{1}{2})||\Sigma(a(N_i;\xi))||_2^2\}; \end{cases} \tag{3}$$

where $\Sigma = [\varepsilon_j]_{n_s \times 1}$ is the residual vector generated by the cycle $N = N_i$, and $||\cdot||_2$ is the Euclidean norm in $\mathbb{R}^{n_s}$. The $j$th entry of this vector is defined by:

$$\varepsilon_j(N_i; \boldsymbol{\xi}) = (\mathcal{P}_a - a)(N_i; \boldsymbol{\xi}_j) = \left[ \sum_{k=1}^{n} a_k(N_i) \psi_k(\boldsymbol{\xi}_j) \right] - a(N_i; \boldsymbol{\xi}_j) = (\Psi(\boldsymbol{\xi}_j))^T \boldsymbol{a}_i - a(N_i; \xi_j). \tag{4}$$