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An asymptotic approach to the adhesion of thin stiff films

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ABSTRACT

In this paper, the asymptotic first order analysis, both mathematical and numerical, of two structures bonded together is presented. Two cases are considered, the gluing of an elastic structure with a rigid body and the gluing of two elastic structures. The glue is supposed to be elastic and to have its stiffness of the same order than those of the elastic structures. An original numerical method is developed to solve the mechanical problem of stiff interface at order 1, based on the Nitsche's method. Several numerical examples are provided to show the efficiency of both the analytical approximation and the numerical method.

1. Introduction

Adhesive bonding is an assembly technique often used in structural mechanics. In bonded composite structures, the thickness of the glue is much smaller than the other dimensions. For example, in Goglio et al. (2008), the glue thickness is 0.1 mm, whereas the dimension of the structure is about 150 mm, giving a typical dimensional ratio close to 1/1500. Thus, the glue thickness can be considered as a small parameter in the modeling process. Usually, the glue stiffness is taken as another small parameter when compared with the adherents stiffness (soft interface theory), as shown in Lebon et al. (2004) and Lebon and Rizzoni (2008). For example, in Goglio et al. (2008), two steel structures are bonded by a Loctite 300 glue and the ratio between the Young moduli of the materials is close to 1/230. Nevertheless, in the case of an epoxy based adhesive bonding of two aluminium structures, the ratio between the Young moduli is typically about 1/20 (see for example Cognard et al. (2011)). Thus, the glue stiffness cannot be considered as the smallest parameter (stiff interface theory). The aim of this paper is to analyze mathematically and numerically the asymptotic behavior of bonded structures in the case of only one small parameter: the thickness. In the following, the stiffness is not a small parameter, and the Young moduli of the glue and of the adherents are of the same order of magnitude.

The mechanical behavior of thin films between elastic adherents was studied by several authors: Abdelmoula et al. (1998), Benveniste (2006), Bertoldi et al. (2007a), Bertoldi et al. (2007b), Bigoni and Movchan (2002), Cognard et al. (2011), Duong et al. (2011), Goglio et al. (2008), Hirschberge et al. (2009), Krasuki and Lenci (2000), Kumar and Mittal (2011), Lebon et al. (2004), Lebon and Rizzoni (2008), Lebon and Rizzoni (2010), Lebon and Rizzoni (2011a), Lebon and Rizzoni (2011), Lebon and Ronel-Idrissi (2007), Nguyen et al. (2012), Rizzoni and Lebon (2012), and Sacco and Lebon (2012). The analysis was based on the classic idea that a thin adhesive film can be replaced by a contact law, like in Abdelmoula et al. (1998). The contact law describes the asymptotic behavior of the film in the limit as its thickness goes to zero and it prescribes the jumps in the displacement (or in the displacement rate) and in the traction vector fields at the limit interface. The limit problem formulation involves the mechanical and the geometrical properties of the adhesive and the adherents, and in Lebon et al. (2004), Lebon and Rizzoni (2008), Lebon and Rizzoni (2010), Lebon and Rizzoni (2011a), Lebon and Rizzoni (2011), Rizzoni and Lebon (2012), and Lebon and Ronel-Idrissi (2007) several cases were considered: soft films (Klarbring (1991) and Lebon et al. (2004)); adhesive films governed by a non-convex energy (Lebon et al. (1997), Lebon and Rizzoni (2008), and Licht and Michaille (1997); imperfect gluing

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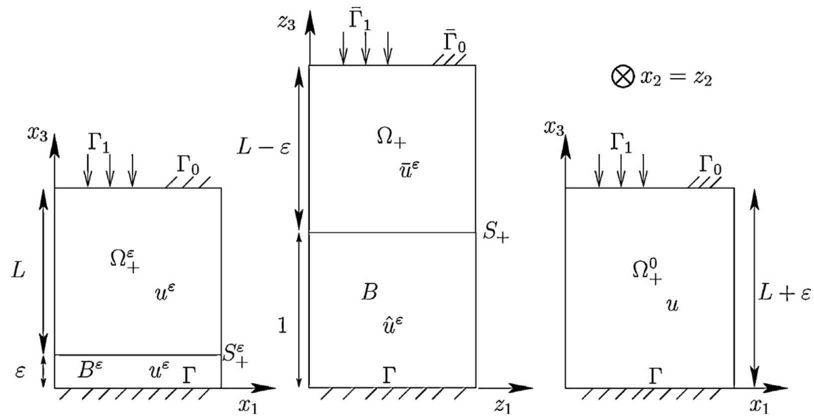

Fig. 1. (a) Initial, (b) rescaled, and (c) limit configuration of a solid glued to a rigid base.

Zaittouni et al. (2002)); flat linear elastic films having stiffness comparable with that of the adherents and giving rise to imperfect adhesion between the films and the adherents (Lebon and Rizzoni (2010) and Lebon and Rizzoni (2011a)); joints with mismatch strain between the adhesive and the adherents, see for example Rizzoni and Lebon (2012). Several mathematical techniques can be used to perform the asymptotic analysis: Γ-convergence, variational analysis, matched asymptotic expansions and numerical studies (see Lebon and Rizzoni (2011) and Sánchez-Palencia (1980) and references therein).

The first part of the paper extends the imperfect interface law given in Lebon and Rizzoni (2011a) to the case of a very thin interphase whose stiffness is of the same order of magnitude as that of the adherents, firstly when an elastic body is glued to a rigid base, and secondly in the plane strain case. In the second part of the paper, numerical methods adapted to solve the limit problems obtained in the first part are developed. In the case of the gluing of a deformable body with a rigid solid, the numerical scheme is very classical. On the contrary, the gluing of two deformable bodies leads to more complicated numerical strategies. The proposed method is based on an original method presented in Nitsche (1974). This kind of method is well known in the domain decomposition context. This method is implemented in a finite element software. In the third part, some numerical examples are presented and the numerical results are analyzed (in terms of mechanical interpretation, computed time, convergence, etc.) in order to quantify and justify the methodology.

2. Theoretical results for thin stiff films

2.1. Asymptotic analysis for an elastic body glued to a rigid base

Let us consider a linear elastic body $\Omega \subset \mathrm{IR}^3$ of boundary $\partial\Omega$. This structure is made of two parts (the adherents) perfectly bonded with a very thin third one (the glue or the interphase), see Fig. 1. Initially, one of the two adherents is considered as rigid. After introducing a small parameter $\varepsilon > 0$ denoting thickness of the glue, we define the following domains:

- $B^\varepsilon = \{(x_1, x_2, x_3) \in \Omega : 0 < x_3 < \varepsilon\}$ (the glue);
- $\Omega_+^\varepsilon = \{(x_1, x_2, x_3) \in \Omega : x_3 > \varepsilon\}$ (the deformable adherent);
- $S_+^\varepsilon = \{(x_1, x_2, x_3) \in \Omega : x_3 = \varepsilon\}$;
- $\Gamma = \{(x_1, x_2, x_3) \in \Omega : x_3 = 0\}$ (the interface);
- $B = \{(x_1, x_2, x_3) \in \Omega : 0 < x_3 < 1\}$;
- $\Omega_+ = \{(x_1, x_2, x_3) \in \Omega : x_3 > 1\}$;
- $S_+ = \{(x_1, x_2, x_3) \in \Omega : x_3 = 1\}$;
- $\Omega_+^0 = \{(x_1, x_2, x_3) \in \Omega : x_3 > 0\}$.

On a part Γ_1 of $\partial\Omega$, an external load g is applied, and on $\Gamma_0 \subset \partial\Omega$, such that $\Gamma_0 \cap \Gamma_1 = \emptyset$, a displacement u_d is imposed. Moreover, we suppose that $\Gamma_0 \cap B^\varepsilon = \emptyset$ and $\Gamma_1 \cap B^\varepsilon = \emptyset$. A body force f is applied in Ω_+^ε. We consider also that the interface Γ is a plane normal to the third direction e_3. We are interested in the equilibrium of such a structure.

The equations of the problem are

$$\begin{cases} \mathrm{div}\sigma^\varepsilon + f = 0 & \text{in} \quad \Omega_+^\varepsilon \cup B^\varepsilon \\ \sigma^\varepsilon n = g & \text{on} \quad \Gamma_1 \\ u^\varepsilon = u_d & \text{on} \quad \Gamma_0 \\ u^\varepsilon = 0 & \text{on} \quad \Gamma \\ \sigma^\varepsilon = A_+ e(u^\varepsilon) & \text{in} \quad \Omega_+^\varepsilon \\ \sigma^\varepsilon = \hat{A} e(u^\varepsilon) & \text{in} \quad B^\varepsilon \end{cases} \tag{1}$$