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Stable extreme damping in viscoelastic two-phase composites with non-positive-definite phases close to the loss of stability

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ABSTRACT

By investigating the effective response of linear viscoelastic composites, we demonstrate that stiff systems can exhibit stable extreme increases in overall damping if one of the composite phases loses positive-definiteness of its elasticities. While non-positive-definite elastic moduli (often referred to as *negative stiffness*) are thermodynamically unstable in unconstrained homogeneous solids, the geometric constraints among constituents in a composite can provide sufficient stabilization. Allowing for negative-stiffness phases in principle expands the range of attainable composite properties and promises extremely high composite stiffness and damping (significantly beyond those of the composite base materials) if the composite is appropriately tuned. This, however, raises questions of stability. In particular, the resulting high damping in stiff composites so far has only been shown to be stable in simple structural and elementary spring-dashpot systems, and therefore has remained a key open question for general composite materials. Studying successively the examples of a spring-dashpot model, a two-phase solid, and a general particle–matrix composite, we demonstrate that a non-positive-definite phase may indeed result in stable extreme damping, which is in line with recent experimental findings.

1. Introduction

Composite materials are a popular means to achieving superior viscoelastic performance, where viscoelasticity characterizes the time-dependent stress–strain response of solids, comprising their stiffness and damping capacity. Due to competing microstructural mechanisms, naturally occurring materials generally exhibit either high stiffness or high damping but not both, see e.g. (Ashby, 1989; Chen and Lakes, 1993). Therefore, one commonly combines high-stiffness phases and lossy high-damping phases in a composite to provide stiff materials with the ability to effectively damp vibrations, for recent examples see e.g. (Kim et al., 2002; Sain et al., 2013; Meaud et al., 2013). This unique but rare combination of high stiffness and high damping is of great interest across technological and scientific disciplines with applications in air- and spacecraft design, scientific instrumentation, seismic protection, or sound and vibration insulation, or generally everywhere stiff and strong members must attenuate vibrations.

If nonlinear effects are neglected as e.g. in the presence of only small stress and strain amplitudes, the solid can be described by a linear viscoelastic model relating stresses and strains through time-dependent material constants (Christensen, 1971). In the special case of harmonic loading, these moduli and therefore the mechanical response become frequency-dependent (Lakes, 1999). In the low-frequency regime, sufficiently far away from the natural frequency, we may neglect inertial effects, so that the linear viscoelastic response of a solid can be obtained by recourse to the correspondence principle (Read, 1950; Lee, 1955): if an equilibrium solution is known for the corresponding linear elastostatic problem, the linear viscoelastic harmonic solution is obtained by replacing the elastic moduli by their complex-valued and frequency-dependent viscoelastic counterparts. The complex viscoelastic moduli in turn allow for the extraction of the dynamic moduli and the corresponding loss factors that characterize the material's damping.

The effective viscoelastic properties of a composite material result from the individual constituent properties and their geometric arrangement and bonding (Milton, 2001). Exact solutions are known only for very few specific cases. In general, bounds on the effective properties have been established for linear elastic (isotropic) composites e.g. by Voigt (1889) and Reuss (1929) or Hashin and Shtrikman (1963), which can be extended to linear viscoelastic solids by use of the correspondence principle (Wang and Lakes, 2005). For positive-definite constituents, these bounds show that the overall properties of a composite can never surpass those of the constituents, thereby greatly limiting the attainable space of stable viscoelastic moduli in composite materials.

Recently, it was shown that the geometric constraints among the phases of a composite can stabilize what is unstable in an unconstrained solid, see e.g. (Drugan, 2007;

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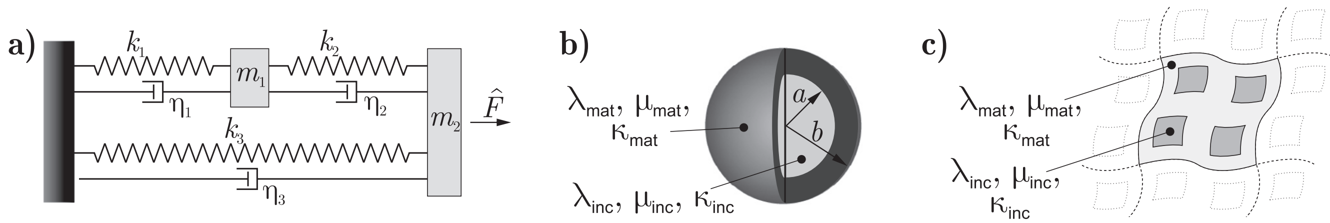

Fig. 1. Schematic of the three model composite systems: (a) system of linear springs, dashpots, and point masses; (b) linear viscoelastic two-phase solid of a coated spherical inclusion; (c) linear viscoelastic particle–matrix composite with periodic or random microstructure.

Kochmann and Drugan, 2009): individual composite constituents may exhibit stable non-positive-definite elastic moduli (so-called *negative stiffness*), which would generally result in mechanical instability in a homogeneous free-standing solid without the composite constraints (Kirchhoff, 1859). This applies equally to two-phase solids of e.g. coated cylinders and coated spheres (Drugan, 2007) as well as to general particle–matrix composites (Kochmann, 2012; Kochmann and Drugan, 2012).

Negative stiffness (i.e. non-positive-definite elastic moduli) can be realized experimentally in solids undergoing structural transitions which render equilibrium states unstable. For example, piezoceramic perovskites in their high-temperature cubic phase become unstable as the temperature is reduced below the Curie point and the tetragonal phase variants produce a stable equilibrium. Stabilizing the high-temperature phase below the Curie point yields an unstable solid with non-positive-definite moduli. This was demonstrated experimentally by Jaglinski et al. (2007) who realized the negative stiffness effect in piezoceramic barium titanate particles (stabilized in a metal-matrix composite) and measured the effective viscoelastic properties. Hence, time-dependent negative stiffness in solids can indeed be realized when viscoelastic solids undergo small-scale mechanical instability. Characterizing their viscoelastic properties experimentally, however, is challenging as this can only be done indirectly e.g. in composite materials containing such phases (due to the delicate stability of negative-stiffness phases which may only exist in a constrained environment).

By allowing for non-positive-definite phases, a review of the attainable composite performance has resulted in predictions of extraordinary mechanical properties: Lakes and Drugan (2002) proposed elastic composites with extremely high effective stiffness; Wang and Lakes (2004) suggested that extremely high stiffness and damping can be achieved (where by *extreme* we imply values that surpass those of the base materials). Various other physical properties have the potential to exhibit anomalies as well, including piezo- or pyroelectricity, or thermal expansion (Wang and Lakes, 2001). Unfortunately, extreme stiffness values due to such negative-stiffness phases in an unconstrained elastic composite are unstable under static conditions (Wojnar and Kochmann, 2013a,b) (but may be stabilized dynamically at high frequency (Kochmann and Drugan, 2011)). Extreme damping due to negative-stiffness components, in contrast, was shown to be stable in spring-dashpot systems (Wang and Lakes, 2004), which are commonly employed to represent linear viscoelastic solids. Furthermore, recent experiments have confirmed considerable damping increases in structural systems due to pre-stressed buckled columns realizing the negative-stiffness effect (Kashdan et al., 2012). Furthermore, using numerical methods and topology optimization, Prasad and Diaz (2009) found specific arrangements of stable particle–matrix composites whose negative-stiffness inclusions are responsible for viscoelastic frequency-softening. Note that such analyses must be handled with caution: positive-definiteness of the resulting effective elasticity tensor of a homogenized composite is generally not sufficient for overall stability, as shown e.g. by Kochmann and Drugan (2012) and Wojnar and Kochmann (2013a), and exact stability conditions have only been derived for few specific composite geometries. Therefore, present understanding of whether or not extreme damping in composites due to non-positive-definite phases can be stable is incomplete for two reasons (among others): (i) besides highly idealized spring-dashpot examples there has been no analysis of actual composites, exploring the full range of stable moduli combinations and examining the possibility of extreme damping; and (ii) effective property predictions based on composite bounds and the correspondence principle neglect inertial effects so that the interesting case of resonant damping in composites with non-positive-definite phases has not been investigated.

Here, we investigate the effective linear viscoelastic response of specific two-phase solids and composites in the subresonant and resonant regime, and we explore the beneficial impact of a non-positive-definite phase on the overall damping and dynamic moduli. Most previous analyses of the viscoelastic behavior of solids having a negative-stiffness phase were based either on simple spring-dashpot models or on bounds on the effective composite moduli. Here, for the first time we analyze the effective dynamic stiffness and damping of two-phase solids using exact continuum-mechanics solutions. Similar examples of two-phase solids were investigated before with respect to their effective linear elastic properties, see e.g. (Kochmann and Drugan, 2009, 2012), while their viscoelastic moduli and specifically their damping capacity have been neglected. The paper is structured as follows: Section 2 explains the employed viscoelastic models as well as the derivation of effective properties and overall stability. The following three sections then apply these models and methods to the specific examples shown in Fig. 1. Section 3 studies a spring-dashpot system to illustrate the phenomena of interest. Section 4 examines the two-phase solid of a coated spherical inclusion, and Section 5 investigates (periodic or random) particle–matrix composites. Finally, Section 6 concludes the analysis.

2. Linear viscoelastic solids

2.1. Constitutive model

Using infinitesimal stress and strain tensors, $\boldsymbol{\sigma}$ and $\boldsymbol{\varepsilon}$, respectively, the stress response $\boldsymbol{\sigma}(t)$ of a linear viscoelastic solid subject to strain history $\boldsymbol{\varepsilon}(t)$ (with t denoting time) can be written as

$$\sigma_{ij}(t) = \int_0^t C_{ijkl}(t-\tau)\,\frac{\partial \varepsilon_{kl}(\tau)}{\partial \tau}\;\mathrm{d}\tau. \qquad (1)$$

Here and in the following we employ standard index notation and the summation convention. $\mathbf{C}(t)$ is the time-dependent fourth-order viscoelastic modulus tensor which is often approximated by a finite number N of relaxation responses with relaxation times τ:

$$\mathbf{C}(t) = \sum_{\beta=1}^{N} \mathbf{C}_\beta \exp\left(-\frac{t}{\tau_\beta}\right). \qquad (2)$$