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Elastic–plastic ductile damage model based on strain-rate plastic potential

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ABSTRACT

Modeling of ductile damage is generally done using analytical potentials, which are expressed in the stress space. In this paper, for the first time it is shown that strain-rate potentials which are exact conjugate of the stress-based potentials can be instead used to model the dilatational response of porous polycrystals. A new integration algorithm is also developed. It is to be noted that a strain-rate based formulation is most appropriate when the plastic flow of the matrix is described by a criterion that involves dependence on all stress invariants. In such cases, although a strain-rate potential is known, the stress-based potential cannot be obtained explicitly. While the proposed framework based on strain-rate potentials is general, for comparison purposes in this work we present an illustration of the approach for the case of a porous solid with von Mises matrix containing randomly distributed spherical cavities. Comparison between simulations using the strain-rate based approach and the classical stress-based Gurson's criterion in uniaxial tension is presented. These results show that the model based on a strain-rate potential predicts the dilatational response with the same level of accuracy.

1. Introduction

Ziegler (1977) has shown that a plastic strain-rate potential can be associated to any convex stress potential. Hence, a strain-rate potential can be used instead of a classical stress potential to describe the plastic response of materials. Strain-rate potentials are more suitable for process design, especially for solving inverse problems (e.g. Chung et al., 1997). Specifically, exact strain-rate potentials associated to the von Mises, Hill (1948), or Cazacu et al. (2006) criteria have been used for metal forming simulations (e.g. Rabahallah et al., 2009a). Barlat and co-workers have also proposed several non-quadratic anisotropic strain-rate potentials (for a review, see Kim et al., 2007). However, at present strain-rate potentials have been used only for the description of the plastic response of fully dense metallic materials (void free materials). For such materials, yielding is insensitive to the mean stress and plastic deformation is not accompanied by any volume change. Therefore, the associated strain-rate potentials are expressed in terms of the deviator of the strain-rate tensor. As a consequence, all the existing time-integration algorithms based on strain-rate potentials make use of the hypothesis that the plastic flow is incompressible. However, most engineering materials contain defects (either cracks or voids). Early on it has been recognized that the presence of defects induces a dependence of the plastic response on the mean stress (Rice and Tracey, 1969; Tvergaard, 1981). To model the particularities of the plastic flow of voided polycrystals, micromechanically-motivated stress-based potentials have been developed. In particular, Gurson's (1977) is the most widely used criterion for modeling yielding of porous metals.

In this paper, it is shown that strain-rate potentials (SRP) can be instead used to numerically model damage–plasticity couplings. Illustration of this approach is done by considering the strain-rate potential which is the exact conjugate of Gurson's (1977) stress-based potential for porous solids containing randomly distributed spherical voids. The structure of the paper is as follows. After a brief presentation of the kinematic homogenization approach of Hill-Mandel (Hill, 1967; Mandel, 1972), we recall Gurson's (1977) analysis and give the expression of the associated SRP (Section 2). The governing equations for an elastic-plastic damage model based on this SRP and the proposed time-integration algorithm are presented in Section 3. The developed algorithm is implemented in the FE code Abaqus/Standard as a user material subroutine (UMAT). For validation purposes, simulations of single-element uniaxial tension using the Abaqus built-in model and the developed UMAT are presented. Furthermore, in order to demonstrate the ability of the new SRP-based model to predict the salient features of ductile damage, an analysis of void volume fraction evolution in a notched tensile bar is conducted. Regarding notations, vectors and tensors are

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denoted by boldface characters. If **A** and **B** are second-order tensors, the contracted tensor product between such tensors is defined as: $\mathbf{A}:\mathbf{B} = A_{ij}B_{ij}$, $i, j = 1\ldots3$. The norm of **A** is defined as $\left\|\mathbf{A}\right\| = \sqrt{\mathbf{A}:\mathbf{A}}$; tr denotes the trace of the tensor.

2. Modeling framework

Generally, the onset of plastic flow is described by specifying a convex yield function, $\varphi(\boldsymbol{\sigma})$, in the stress space and the associated flow rule

$$\mathbf{D}^{\mathrm{p}} = \dot{\lambda}\frac{\partial\phi}{\partial\boldsymbol{\sigma}} \tag{1}$$

where $\boldsymbol{\sigma}$ is the Cauchy stress tensor, $\mathbf{D}^{\mathrm{p}}$ denotes the plastic strain-rate tensor and $\dot{\lambda}\geq 0$ stands for the plastic multiplier. The yield surface is defined as $\varphi(\boldsymbol{\sigma}) = \tau$, where τ is a positive scalar with the dimension of stress. Generally, τ is taken as the uniaxial yield in tension, σ_T. The dual potential of the stress potential $\varphi(\boldsymbol{\sigma})$ is defined (see Ziegler (1977), Hill (1987)) as:

$$\psi(\mathbf{D}^{\mathrm{p}}) = \dot{\lambda}, \tag{2}$$

and

$$\boldsymbol{\sigma} = \sigma_T\frac{\partial\psi}{\partial\mathbf{D}^{\mathrm{p}}} \tag{3}$$

The yield function $\varphi(\boldsymbol{\sigma})$ is generally taken homogeneous of degree one with respect to positive multipliers, so

$$\dot{W}^p = \sup_{\boldsymbol{\sigma}\in\mathbb{C}}(\sigma_{ij}D^p_{ij}) = \dot{\lambda}\sigma_T,\ i, j = 1\ldots3, \tag{4}$$

where $\mathbb{C}$ is the convex domain delimited by the yield surface, and $\dot{W}^p$ is the work rate associated with the plastic strain-rate tensor $\mathbf{D}^p$. Thus, $\Psi(\mathbf{D}^{\mathrm{p}})$ is a work-equivalent measure of the strain-rate. The functions $\Psi(\mathbf{D}^{\mathrm{p}})$ and $\varphi(\boldsymbol{\sigma})$ are dual potentials. For example, in the case of von Mises potential (i.e. $\varphi(\boldsymbol{\sigma}) = \sqrt{(3/2)\boldsymbol{\sigma}':\boldsymbol{\sigma}'}$), the associated SRP is: $\psi(\mathbf{D}^{\mathrm{p}}) = \sqrt{(2/3)\mathbf{D}^{\mathrm{p}}:\mathbf{D}^{\mathrm{p}}} = \dot{\bar{\varepsilon}}$, where $\dot{\bar{\varepsilon}}$ denotes the von Mises equivalent strain-rate and $\boldsymbol{\sigma}'$ the stress deviator.

2.1. Plastic potentials for porous metallic materials

The kinematic homogenization approach of Hill–Mandel (Hill, 1967; Mandel, 1972) offers a rigorous framework for the development of criteria for describing the plastic response of porous solids. If the matrix is rigid-plastic, it has been shown (e.g. Talbot and Willis, 1985) that there exists a strain-rate potential $\Pi = \Pi(\mathbf{D}^p, f)$ such that the stress at any point in the porous solid is given by:

$$\boldsymbol{\sigma} = \frac{\partial\Pi(\mathbf{D}^{\mathrm{p}}, f)}{\partial\mathbf{D}^{\mathrm{p}}}\ \text{with}\ \ \Pi(\mathbf{D}^{\mathrm{p}}, f) = \inf_{\mathbf{d}\in K(\mathbf{D})}\left\langle\pi(\mathbf{d})\right\rangle_{\Omega} \tag{5}$$

where Ω is a representative volume element composed of the matrix and a traction-free void, while $\langle\rangle$ denotes the average value over Ω; f is the porosity (ratio between the volume of the void and the volume of Ω); $\pi(\mathbf{d})$ is the matrix's plastic dissipation with $\mathbf{d}$ being the local plastic strain-rate tensor. Minimization is done over $K(\mathbf{D})$, which is the set of incompressible velocity fields compatible with homogeneous strain-rate boundary conditions, i.e.

$$\mathbf{v} = \mathbf{Dx}\ \text{for any}\ \mathbf{x}\in\partial\Omega \tag{6}$$

Only very few velocity fields compatible with uniform strain-rate boundary conditions are known. For example, for spherical void geometry the only known velocity fields are those deduced by Rice and Tracey (1969) and Budyanski et al. (1982). For examples of other velocity fields deduced using an Eshelby-type approach, the reader is referred to Monchiet et al. (2011). Furthermore, in order to arrive at closed-form expressions, the local plastic dissipation is calculated for a unique velocity field. Thus, the associated overall potential is an upper-bound of the exact plastic dissipation of the porous solid. However, only in the case when the plastic behavior of the matrix is described by simple expressions (e.g. von Mises, Hill, 1948), it is possible to arrive at a closed-form expression of the approximate stress-based plastic potentials of the porous solid (e.g. Gurson, 1977; Monchiet et al., 2008, respectively). If the plastic flow of the matrix is described by a criterion involving all stress invariants, e.g. Tresca criterion, an approximate SRP can be deduced (see Appendix A); however, a closed-form stress-based criterion can be obtained only in parametric form (see Cazacu et al., 2013). Furthermore, integration algorithms exist only for stress-based formulations of coupled elasto-plastic damage behavior (e.g. Aravas, 1987). Although all the numerical methods and techniques developed in this paper are valid for an elasto-plastic damage model described by a general strain-rate potential $\Pi = \Pi(\mathbf{D}^p, f)$ in its general form, in this paper we discuss a specific strain-rate potential which is the exact conjugate of Gurson's (1977) stress potential for spherical cavities. Let us recall that the analysis of Gurson (1977) was done on a hollow-sphere, its rigid-plastic behavior being governed by the von Mises yield criterion. The local plastic dissipation was calculated using the velocity field deduced by Rice and Tracey (1969). The approximate strain-rate potential obtained is:

$$\Psi\left(\mathbf{D}^{\mathrm{p}}, f\right) = 2\left|D^p_m\right|\left[\frac{\sqrt{1+u^2}-\sqrt{f^2+u^2}}{u} + \ln\left(\frac{u+\sqrt{f^2+u^2}}{u+\sqrt{1+u^2}}\frac{1}{f}\right)\right], \tag{7}$$

where f denotes the porosity (or void volume fraction), $u = 2\left(\left|D^p_m\right|/D^p_e\right)$, with $D^p_m = (tr\,\mathbf{D}^p)/3$ and $D^p_e = \sqrt{(2/3)\mathbf{D}'^p:\mathbf{D}'^p}$ Hence, at yielding:

$$\frac{\sigma_m}{\sigma_T} = \frac{1}{3}\frac{\partial\Psi\left(\mathbf{D}^p, f\right)}{\partial D^p_m} = \frac{2}{3}\ln\left(\frac{u+\sqrt{u^2+f^2}}{u+\sqrt{u^2+1}}\cdot\frac{1}{f}\right), \tag{8a}$$

$$\frac{\sigma_e}{\sigma_T} = \left|\frac{\partial\Psi\left(\mathbf{D}^p, f\right)}{\partial D^p_e}\right| = \sqrt{1+u^2}-\sqrt{u^2+f^2}, \tag{8b}$$

where $\sigma_m = tr(\boldsymbol{\sigma})/3$ and $\sigma_e = \sqrt{(3/2)\boldsymbol{\sigma}':\boldsymbol{\sigma}'}$. The parameter u can be eliminated between Eq. (8a) and (8b), to arrive at the classical stress-based formulation (for details, see Gurson, 1977):

$$\Phi(\boldsymbol{\sigma}, f) = \left(\frac{\sigma_e}{\sigma_T}\right)^2 + 2f\cosh\left(\frac{3\sigma_m}{2\sigma_T}\right) - 1 - f^2. \tag{9}$$

As an example, in Fig. 1(a) is shown the representation of the strain-rate potential (7) for different initial porosities $f = 0.001$, $f = 0.01$, and $f = 0.1$, respectively, while in Fig. 1(b) are shown iso-contours of its exact dual, i.e. Gurson's stress potential (Eq. (9)) for the same porosities. The porous material being isotropic, the principal directions of $\mathbf{D}^{\mathrm{p}}$ and stress coincide. The projection of the strain-rate potential in the octahedral plane (plane with normal at equal angles to the principal directions of the strain-rate tensor $\mathbf{D}^{\mathrm{p}}$) is shown in Fig. 2(a) while Fig. 2(b) depicts the section of Gurson's (1977) stress potential.

3. Time-integration algorithm for a general elastic–plastic damage model based on a strain-rate plastic potential

In the following we present the governing equations for an elastic-plastic damage model based on a strain-rate potential and a general time-integration algorithm. The total rate of deformation is considered to be the sum of an elastic part and a plastic part $\mathbf{D}^{\mathrm{p}}$. The