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A new formulation of the effective elastic–plastic response of two-phase particulate composite materials

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ABSTRACT

In this paper the double-inclusion model, originally developed to determine effective linear elastic properties of composite materials, is reformulated and extended to predict the effective nonlinear elastic–plastic response of two-phase particulate composites reinforced with spherical particles. The resulting problem of elastic–plastic deformation of a double-inclusion embedded in an infinite reference medium subjected to an incrementally applied far-field strain is solved by the finite element method. The proposed double-inclusion model is evaluated by comparison of the model predictions to the available exact results obtained by the direct approach using representative volume elements containing many particles. It is found that the double-inclusion formulation is capable of providing accurate prediction of the effective elastic–plastic response of two-phase particulate composites at moderate particle volume fractions.

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1. Introduction

Prediction of the effective elastic–plastic response of particulate composite materials consisting of brittle particles and ductile matrix, such as metal matrix composites reinforced with ceramic particles, is an important and active research topic (see, e.g., Pierard et al., 2007). Result of prediction is often presented in the form of stress–strain relation under some simple loading, say, simple tension or pure shear.

Existing methods for predicting the elastic–plastic response of composite materials include the secant homogenization method (Tandon and Weng, 1988), incremental homogenization method based on the Mori–Tanaka model (Doghri and Quaar, 2003), direct approach using representative volume elements (RVEs) (Gonzalez et al., 2004), and periodic unit cell method (Bao et al., 1991). The secant method is limited to monotonic and proportional loading. The incremental homogenization method has no such limitation and can be applied to load reversal or cyclic load. However, in its original form the incremental approach over-predicts elastic–plastic stress–strain response, and the remedy is to use only the isotropic part of the anisotropic elastic–plastic tangent stiffness tensor (Doghri and Quaar, 2003). However, the use of only the isotropic part of the tangent stiffness tensor, while resulting in much improved prediction, lacks either theoretical or physical basis. In addition, fitting parameters may be needed in formulating the isotropic part of the tangent stiffness tensor (Delannay et al., 2007). It is well known that for a particulate composite with its matrix material characterized by the von Mises yield condition (the theory of J_2 plasticity), reinforced with homogeneous, isotropic and linearly elastic particles, the composite as a whole may yield under hydrostatic stress even though the matrix does not (Chu and Hashin, 1971; Qiu and Weng, 1992; Sun and Ju, 2001). One somewhat overlooked fault of the incremental Mori–Tanaka method is its inability to predict the yield of such a composite under hydrostatic stress when the particles are spherical. The direct approach using RVEs gives rigorous prediction of the effective composite elastic–plastic response, but is computationally expensive, particularly given the nonlinear nature of plastic deformation. The unit cell method applies to composites with periodic microstructures. It cannot be rigorously applied to real composites which in general are not periodic. Because of its simplicity the unit cell method is nonetheless often used to approximate the elastic–plastic behavior of real composites (Farrissey et al., 1999; LLorca and Segurado, 2004). In addition to the aforementioned methods, Sun and Ju (2004) applied the ensemble averaging approach to the prediction of the effective elastic–plastic response of particulate composites. However, their method invariably predicts isotropic hardening, and therefore is unable to account for the Bauschinger effect. In this study, the double-inclusion model, originally developed by Hori and Nemat-Nasser (1993) to determine effective linear elastic properties of composite materials (also see Aboutajeddine and Neale, 2005), is reformulated and extended to predict the effective elastic–plastic response of two-phase particulate composite materials.

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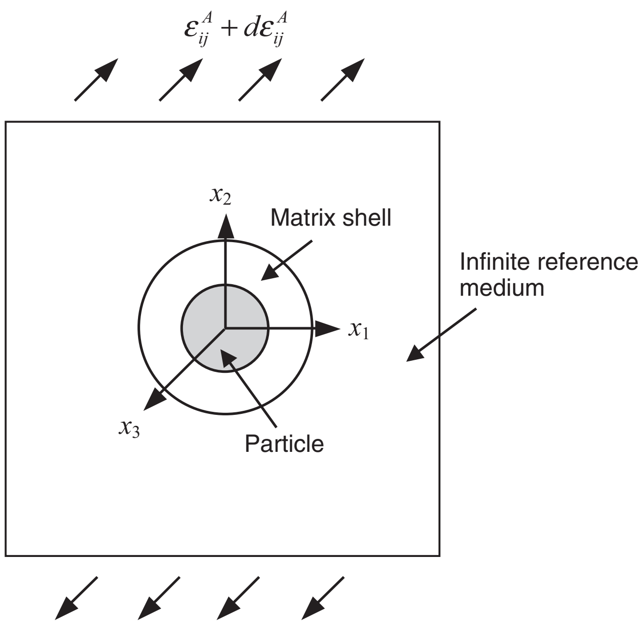

Fig. 1. The double-inclusion model with the origin of the Cartesian coordinate system (x_1, x_2, x_3) at the center of the particle.

It should be mentioned that the majority of published results for elastic–plastic stress–strain relations of composite materials are those under monotonically increasing uniaxial tension. Composite materials in general exhibit the Bauschinger effect even though the constituent materials undergo only isotropic strain hardening. To characterize the Bauschinger effect, it is necessary to determine stress–strain relation under a loading path that includes loading in tension followed by unloading and reverse loading in compression.

2. Formulation

Consider a particle-reinforced composite material. It is assumed that the particles are spherical, the particles and the matrix are homogeneous, isotropic and elastic–plastic, and the particles and the matrix are perfectly bonded at their interface. Small deformation is also assumed. To determine the effective elastic–plastic response of the composite, one can consider the elastic–plastic deformation of a representative volume element of the composite under continued loading by means of boundary surface displacement or traction. Define a Cartesian coordinate system (x_1, x_2, x_3) in connection with the representative volume element. At $t = 0$, where t is time (or more precisely a time-like monotonically increasing variable), the composite is in the initial undeformed configuration. At t the composite representative volume element is subjected to a boundary surface displacement u_i^0 or traction T_i^0, and the boundary displacement or traction at $t + dt$ with $dt > 0$ can then be written as $u_i^0 + du_i^0$ or $T_i^0 + dT_i^0$ $(i = 1, 2, 3)$, where du_i^0 or dT_i^0 is the boundary displacement increment or traction increment at t. The stress in the composite at t is denoted by σ_{ij} and the strain by ε_{ij} $(i, j = 1,2,3)$. The stress in the composite at $t + dt$ is denoted by $\sigma'_{ij} = \sigma_{ij} + d\sigma_{ij}$ and the strain by $\varepsilon'_{ij} = \varepsilon_{ij} + d\varepsilon_{ij}$, where $d\sigma_{ij}$ and $d\varepsilon_{ij}$ are the stress and strain increment, respectively, resulting from the boundary displacement increment du_i^0 or traction increment dT_i^0.

The strain in the composite at $t + dt$ is then given by

$$\varepsilon'_{ij} = \int_0^{t+dt} d\varepsilon_{ij} \tag{1}$$

where the integral is taken over the strain path. The volume average strain increment of the composite is given by

$$d\bar{\varepsilon}_{ij} = \frac{1}{V}\int_V d\varepsilon_{ij} dV \tag{2}$$

where V is the volume of the composite RVE. The average strain of the composite at $t + dt$ is defined as

$$\bar{\varepsilon}'_{ij} = \int_0^{t+dt} d\bar{\varepsilon}_{ij} \tag{3}$$

where the integration is taken over the given strain path.

The volume average stresses of the composite $\bar{\sigma}_{ij}$ and $\bar{\sigma}'_{ij}$ at t and $t + dt$, respectively, are defined as follows.

$$\bar{\sigma}_{ij} = \frac{1}{V}\int_V \sigma_{ij} dV \tag{4a}$$

$$\bar{\sigma}'_{ij} = \frac{1}{V}\int_V \sigma'_{ij} dV \tag{4b}$$

The average stress increment is $d\bar{\sigma}_{ij} = \bar{\sigma}'_{ij} - \bar{\sigma}_{ij}$.

The yield criterion of either the particle or matrix material of the composite is expressed in the form $f(\sigma_{ij}) = 0$, where $f(\sigma_{ij}) \equiv f(\sigma_{11}, \sigma_{12}, \sigma_{13}, \sigma_{21}, \sigma_{22}, \sigma_{23}, \sigma_{31}, \sigma_{32}, \sigma_{33})$ is the yield function. If a constituent material of the composite obeys the von Mises yield criterion with isotropic strain hardening, the yield function is given by $f(\sigma_{ij}) = \sigma_{eq} - Y(\varepsilon_{eq}^p)$, where σ_{eq} is the equivalent stress of the constituent material defined as $\sigma_{eq} = (3/2\, s_{ij}\, s_{ij})^{1/2}$, with s_{ij} being the deviatoric stress, and ε_{eq}^p is the equivalent plastic strain of the constituent material defined by the integral $\varepsilon_{eq}^p = \int d\varepsilon_{eq}^p$ over the strain path, with $d\varepsilon_{eq}^p = (2/3\, d\varepsilon_{ij}^p\, d\varepsilon_{ij}^p)^{1/2}$ being the equivalent plastic strain increment. Note that a repeated index indicates summation.

To determine the effective elastic–plastic response of the particulate composite, let the representative volume element be subjected to a boundary surface displacement, with the boundary surface displacement increment at t being a homogeneous displacement increment given by $du_i^0 = d\varepsilon_{ij}^0 x_j (i, j = 1, 2, 3)$, where x_i are the (Cartesian) coordinates of a material point on the boundary surface of the RVE at t, and $d\varepsilon_{ij}^0$ is a uniform strain increment. Note that $d\varepsilon_{ij}^0$ can be different at different time t. The volume average strain increment of the composite then becomes $d\bar{\varepsilon}_{ij} = d\varepsilon_{ij}^0$. The average strain of the composite at $t + dt$ is given by

$$\bar{\varepsilon}'_{ij} = \int_0^{t+dt} d\varepsilon_{ij}^0 \tag{5}$$

Therefore, the average strain of the composite can be determined simply by integrating the applied uniform strain increment $d\varepsilon_{ij}^0$ over the prescribed strain path. With the direct approach the corresponding average stress can be found in principle by solving the stress field in the representative volume element by using, say, the finite element method. Given the nonlinear nature of elastic–plastic deformation and that a representative volume element usually contains a large number of particles, the solution of the stress field is a computationally formidable problem. An alternative to the direct approach is to evaluate the volume average stress of the composite using micromechanics homogenization models, for example, the incremental Mori–Tanaka model. In this paper a new approach based on the double-inclusion model is proposed to determine the effective elastic–plastic response of a particulate composite.

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