Mechanics Research Communications 52 (2013) 101–107

Contents lists available at ScienceDirect

Mechanics Research Communications

journal homepage: www.elsevier.com/locate/mechrescom

On shear thinning fluid flow induced by continuous mass injection in porous media with variable conductivity

Valentina Ciriello[a,1], Sandro Longo[b,2], Vittorio Di Federico[a,*]

[a] Dipartimento di Ingegneria Civile, Chimica, Ambientale e dei Materiali (DICAM), Università di Bologna, Viale Risorgimento 2, 40136 Bologna, Italy
[b] Dipartimento di Ingegneria Civile, Ambiente Territorio e Architettura (DICATeA), Università di Parma, Parco Area delle Scienze 181/A, 43124 Parma, Italy

ARTICLE INFO

Article history:
Received 12 June 2013
Received in revised form 8 July 2013
Accepted 10 July 2013
Available online 19 July 2013

Keywords:
Non-Newtonian
Porous medium
Injection
Similarity solution
Variable permeability

ABSTRACT

A new formulation is proposed to examine the propagation of the pressure disturbance induced by the injection of a time-variable mass of a weakly compressible shear thinning fluid in a porous domain with generalized geometry (plane, radial, or spherical). Medium heterogeneity along the flow direction is conceptualized as a monotonic power-law permeability variation. The resulting nonlinear differential problem admits a similarity solution in dimensionless form which provides the velocity of the pressure front and describes the pressure field within the domain as a function of geometry, fluid flow behavior index, injection rate, and exponent of the permeability variation. The problem has a closed-form solution for an instantaneous injection, generalizing earlier results for constant permeability. A parameter-dependent upper bound to the permeability increase in the flow direction needs to be imposed for the expression of the front velocity to retain its physical meaning. An example application to the radial injection of a remediation agent in a subsurface environment demonstrates the impact of permeability spatial variations and of their interplay with uncertainties in flow behavior index on model predictions.

1. Introduction

The mechanics of non-Newtonian fluid flow in porous media has attracted substantial attention in the past and present, as many fluids injected in natural or man-made porous formations exhibit a complex rheological nature (Savins, 1969; Barenblatt et al., 1990; Shenoy, 1995; Chhabra et al., 2001; Sochi, 2010). The range of applications spans extraction of crude oils, underground oil displacement, well drilling, aquifer contamination, soil remediation. The specific nature of the fluid involved governs the choice of a constitutive equation describing the relationship between stress and strain; often the fluid rheological behavior is properly described by a Cross or Carreau–Yasuda model with multiple parameters, but can be approximated in a certain range of shear rates by the simpler two-parameter Ostwald–DeWaele model. At the other end of the spectrum, flow in porous media of viscoelastic liquids or suspensions of long molecular particles entails the adoption of more complex models such as Rivlin–Ericksen second grade (Jordan and Puri, 2003) or dipolar fluids (Puri and Jordan, 2006, and references therein). Adoption of a rheological model at the fluid mechanics scale results in a relationship between pressure gradient and specific flux at the Darcy scale. For power-law fluids, this macroscopic motion equation is particularly simple, and reduces to a nonlinear modification of Darcy's law, often derived using a capillary representation of the porous medium (Cristopher and Middleman, 1965; Teeuw and Hesselink, 1980; Pascal and Pascal, 1985; Pearson and Tardy, 2002). Coupling the motion equation with mass balance yields a transient nonlinear advection–diffusion equation, whose solution gives the pressure in the domain of interest. A particular subclass of analytical solutions to these nonlinear problems is derivable upon adopting a self-similar transformation (Barenblatt, 1996), which yields elegant closed-form results for infinite porous domains subject to different initial and boundary conditions. For free-surface, gravity-driven flow in porous media, examples of this approach are Pascal and Pascal (1993), Bataller (2008), Di Federico et al., 2012a,b. For confined flow, Pascal (1991a,b) studied the pressure perturbations generated in an infinite homogeneous porous domain by an instantaneous mass injection in plane or axisymmetric geometry. Their solution was later extended to spherical geometry by Di Federico and Ciriello (2012), while performing a sensitivity analysis on the results.

The objective of this study is to extend the solution developed by Di Federico and Ciriello (2012) in two respects: (i) allowing for a time variable fluid injection as opposed to an instantaneous one, the former being more suitable in many instances to represent e.g. the continuous release of a displacing fluid, environmental contaminant or remediation agent in the subsurface; (ii) to take into account the combined effect of domain heterogeneity by allowing the porous medium permeability to monotonically vary with the

* Corresponding author. Tel.: +39 051 2093750; fax: +39 051 6448346.
E-mail addresses: valentina.ciriello3@unibo.it (V. Ciriello), sandro.longo@unipr.it (S. Longo), vittorio.difederico@unibo.it (V. Di Federico).
[1] Tel.: +39 051 2093753.
[2] Tel.: +39 0521 905157.

0093-6413/$ – see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.mechrescom.2013.07.006

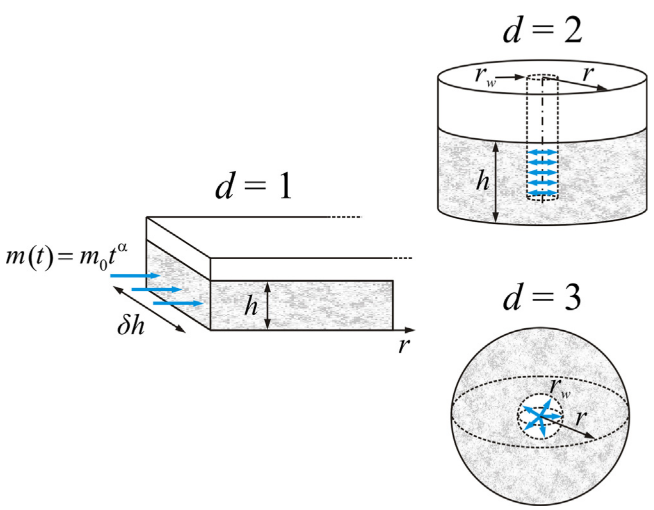

Fig. 1. Domain schematic for plane (d = 1), cylindrical (d = 2), and spherical (d = 3) geometry.

distance from the injection point; such an assumption may represent flow in a domain in which previous injection or pumping has gradually modified the permeability field around the release point in a systematic way. The insight gained may be employed in numerical models of non-Newtonian fluid flow at different scales.

The exposition is organized as follows. The mathematical problem is formulated in Section 2 for a generalized geometry, and solved in Section 3 in self-similar form. Section 4 discusses the limits existing on problem parameters by virtue of formulated assumptions, and their relationship. An application involving the migration of a remediation agent in a subsurface domain is presented in Section 5. Concluding remarks are formulated in Section 6.

2. Problem formulation

A non-Newtonian fluid is injected into an infinite porous domain, initially saturated by another ambient fluid. The propagation of pressure within the domain is taken to be a one-dimensional transient process in plane (d = 1), cylindrical (d = 2) and spherical (d = 3) geometry (Fig. 1). The mass of intruding fluid, injected in the domain origin starting at time t = 0, increases with time as m_0t^α, with m_0 (dimensions $[MT^{-\alpha}]$) and α being constants; α = 0 corresponds to the instantaneous release of a given mass, α = 1 to a constant flux. The domain is described geometrically by its thickness h for d = 1, 2, and by the surface of the injection zone, equal to δh^2 for d = 1 (δ being the aspect ratio of the rectangular injection area), $2\pi h r_w$ for d = 2, and $4\pi r_w^2$ for d = 3, with r_w being the radius of the injecting well for d = 2, 3.

The permeability of the porous domain varies in the direction of propagation according to

$$k(x) = k_0\left(\frac{r}{r^*}\right)^\beta, \tag{1}$$

k_0 being the reference permeability at the length scale r^* and β a real number (Mathunjwa and Hogg, 2007); for β = 0 the porous domain has homogeneous permeability k_0, while for $\beta > 0$ and $\beta < 0$ the permeability respectively increases or decreases with distance from the injection point. A permeability decreasing with distance from the injection well was considered by Altunkaynak and Sen (2011), while for viscous gravity currents in channels of given shape, the channel width was allowed to vary with distance from the source according to a relation akin to (1), to represent widening or narrowing channels (Takagi and Huppert, 2008). The injected fluid is described by the rheological power-law model, given for simple shear flow by $\tau = \tilde{\mu}\dot{\gamma}|\dot{\gamma}|^{n-1}$, in which τ is the shear stress, $\dot{\gamma}$ the shear rate, $\tilde{\mu}$ the fluid consistency index and n the flow behavior index (a positive real number); $n < 1$ represents shear thinning, $n > 1$ shear thickening behavior. The equation of motion for the fluid is a nonlinear modification of Darcy's law, verified experimentally by Cristopher and Middleman (1965) and Yilmaz et al. (2009). Flow and continuity equation read respectively (gravity effects are neglected in spherical geometry):

$$v = \left(-\frac{k(r)}{\mu_{ef}}\frac{\partial p}{\partial r}\right)^{1/n}, \tag{2}$$

$$\frac{1}{r^{d-1}}\frac{\partial}{\partial r}\left(r^{d-1}v\right) = -c_0\phi\frac{\partial p}{\partial t}, \tag{3}$$

where r is the spatial coordinate, t the time, v the Darcy velocity, p the pressure, $c_0 = c_f + c_p$ the total compressibility coefficient, c_f the fluid compressibility coefficient, c_p the porous medium compressibility coefficient, ϕ the porosity, k the permeability coefficient, and μ_{ef} the effective viscosity, given by (Shenoy, 1995)

$$\frac{k}{\mu_{ef}} = \frac{1}{2\tilde{\mu}C_t}\left(\frac{n\phi}{3n+1}\right)^n\left(\frac{50k}{3\phi}\right)^{(n+1)/2}, \tag{4}$$

where $C_t = C_t(n)$ denotes a tortuosity factor for which different expressions are available; in the following, the expression proposed by Pascal and Pascal (1985), i.e. $C_t = \left(25/12\right)^{(n+1)/2}$, will be adopted.

Substituting Eq. (2) in Eq. (3) one obtains:

$$\frac{1}{r^{d-1}}\frac{\partial}{\partial r}\left(r^{d-1}\left(-\frac{\partial p}{\partial r}\right)^{1/n}\right) = -c_0\phi\left(\frac{\mu_{ef}}{k}\right)^{1/n}\frac{\partial p}{\partial t}, \tag{5}$$

with initial condition (p_0 is the ambient pressure)

$$p(r, t=0) = p_0, \tag{6}$$

while conservation of mass $m(t)$ released into the domain requires

$$m(t) = m_0t^\alpha = \omega h^{3-d}\rho\phi\cdot c_0\int_0^{r_N(t)}(p-p_0)\cdot r^{d-1}\,dr, \tag{7}$$

where ρ is fluid density. In Eq. (7), the factor ω takes the values δ for plane, 2π for radial, and 4π for spherical geometry (d = 1, 2, 3 respectively) and $r_N(t)$ denotes the advancing compression front. Under the validity of (3), i.e. moderately compressible fluids (for large compressibility coefficients, additional terms arise, see Pascal and Pascal, 1990), it has been demonstrated earlier for a variety of boundary conditions and a homogeneous domain (Pascal and Pascal, 1985; Di Federico and Ciriello, 2012; Ciriello and Di Federico, 2012) that for shear thinning fluids, the front has finite velocity $u(t) = \phi dr_N/dt$, while for Newtonian and shear thickening fluids, no pressure front exists, and $r_N(t) \to \infty$ for any t. Hence for $n < 1$ the appropriate boundary conditions are

$$p(r_N(t), t) = p_0, \tag{8}$$

$$\left(\frac{\partial p}{\partial r}\right)(r = r_N(t)) = 0, \tag{9}$$

$$r_N(0) = 0. \tag{10}$$

Dimensionless variables are then defined as follows:

$$(R, H, R_N, T, P, P_0, V, U, M_0) = \left(\frac{r}{r^*}, \frac{h}{r^*}, \frac{r_N}{r^*}, \frac{t}{t^*}, \frac{p}{p^*}, \frac{p_0}{p^*}, \frac{vt^*}{r^*}, \frac{ut^*}{r^*}, \frac{m_0t^{*\alpha}}{\rho r^{*3}}\right), \tag{11}$$