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The interaction of an edge dislocation with an inhomogeneity of arbitrary shape in an applied stress field

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ABSTRACT

A general, approximate solution is presented for an edge dislocation interacting with an inhomogeneity of arbitrary shape under combined dislocation and applied stress fields. The solution shows that the contributions of the dislocation stress field and the applied stress field to the interaction follow a simple superposition principle. The dislocation stress field has a short range effect, while the applied stress field has a long range effect. As special cases, explicit solutions for some common inhomogeneity shapes are obtained for the interaction induced by the applied stress field.

1. Introduction

It is well-known that the mobility of dislocations plays an important role in the understanding of the macro strain hardening behavior and micro damage mechanisms of crystalline materials, and this mobility is strongly influenced by the interaction between dislocation and inhomogeneity in the materials. Therefore, the study of interaction between dislocation and inhomogeneity has received a great deal of attention during the last few decades. Most of the studies were traditionally based on solution of appropriate boundary value problems in the linear theory of elasticity. However, due to the complexity of the elastic boundary and interfacial problems only a handful of analytical solutions can be found for highly idealized inhomogeneity shapes, such as circular (Dundurs and Mura, 1964; Dundurs and Gangadharan, 1967; Wang and Pan, 2010, 2011; Fang and Liu, 2006a,b) and elliptical inhomogeneities (Stagni and Lizzio, 1983; Santare and Keer, 1986; Gong and Meguid, 1994; Qaissaunee and Santare, 1995), and a surface layer (Weeks et al., 1968).

Eshelby inclusion theory (Eshelby, 1956, 1961) provides an alternative method to solve the problem. According to this theory, if a homogenous inclusion (the inclusion has the same elastic behavior as the matrix) undergoes a stress-free transformation strain in an applied stress field, the interaction force between the homogenous inclusion and the applied stress field can be determined from the work done by the stress field during the transformation. For an inhomogeneity, it may be transformed to a homogenous one with an equivalent transformation strain on the base of the Eshelby equivalent inclusion theory (Eshelby, 1961; Withers et al., 1989; Li et al., 2011; Zhou et al., 2011). Consequently, the interactions between the stress field and the inhomogeneity can be evaluated by the same method. Based on this approach some approximate solutions for the interaction of an inhomogeneity of arbitrary shape with dislocation have been obtained (Li and Shi, 2002; Shi and Li, 2003). However, these solutions are limited to the inhomogeneity interacting with dislocation stress field only. It is evident that the interaction between dislocation and inhomogeneity in an applied stress field is of more practical importance because the macro strain hardening behavior and micro damage mechanisms of the materials have to be known just under such conditions.

In this study, on the basis of Eshelby inhomogeneity theory, a general, approximate analytical solution is developed for an edge dislocation interacting with an inhomogeneity of arbitrary shape under combined dislocation and remotely applied stress fields. The solution shows that the contributions of the dislocation stress field and the applied stress field to the interaction follow a simple superposition principle. The dislocation stress field becomes dominant to the interaction only when the distance between dislocation and inhomogeneity approaches to nanometer scale, i.e., the dislocation stress field has only a short range effect, while the applied stress field has a long range effect. In view of the fact that the interaction between dislocation and inhomogeneity induced by the dislocation stress field has been extensively studied, in the present paper we focus our attention on the interaction produced by the remotely applied stress field.

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2. Model and formulation

Fig. 1 shows the physical problem to be studied. A straight edge dislocation is located at the origin of a Cartesian coordinate system ($o-xyz$), and the dislocation line coincides with the z-axis. Consider a two-dimensional (2D) inhomogeneity of arbitrary shape (domain Ω) embedded in an infinite matrix and subjected to the dislocation stress field σ_{ij}^d and remotely applied stress field σ_{ij}. This 2D implementation conveys the essence of 3D problem, and represents an inhomogeneity extending throughout the thickness, consistently with experimental observations in thin metal film. According to the Eshelby theory (Eshelby, 1961), the inhomogeneity will undergo an equivalent transformation strain $\mathbf{e^T}$ induced by combined action of the dislocation stress field and the applied stress field. Now consider a differential element dA within the inhomogeneity. The transformation strain in dA can be expressed by Eshelby (1961) and Withers et al. (1989)

$$\mathbf{e^T} = [(\mathbf{C}_i - \mathbf{C_m})\mathbf{S} + \mathbf{C_m}]^{-1}(\mathbf{C_m} - \mathbf{C_i})\mathbf{e}, \tag{1}$$

where $\mathbf{S}$ is the Eshelby tensor, dependent solely upon the shape of the inhomogeneity and the Poisson's ratio ν of the matrix material. $\mathbf{C_i}$ and $\mathbf{C_m}$ are the elastic tensors of the inhomogeneity and the matrix material, respectively. $\mathbf{e}$ is the combined strain field of the edge dislocation field and the applied strain field in the absence of the inhomogeneity. For plane strain condition, the non-zero components of the combined strain $\mathbf{e}$ are given by

$$\left.\begin{aligned} e_{11} &= \frac{\sigma_{11} - \nu(\sigma_{11}+\sigma_{22})}{2\mu_m} - \frac{b(1-2\nu)\sin\theta}{4\pi r(1-\nu)} \\ e_{22} &= \frac{\sigma_{22} - \nu(\sigma_{11}+\sigma_{22})}{2\mu_m} - \frac{b(1-2\nu)\sin\theta}{4\pi r(1-\nu)} \\ e_{12} &= \frac{\sigma_{12}}{\mu_m} + \frac{b\cos\theta}{4\pi r(1-\nu)} \end{aligned}\right\}, \tag{2}$$

in which b is the Burger's vector of the dislocations, μ_m, ν are the shear modulus and Poisson's ratio of the matrix material. The first and second terms in Eq. (2) are, respectively, the applied strains and dislocation strains. As shown in Eq. (1), the equivalent transformation strain $\mathbf{e^T}$ is not zero for an inhomogeneity ($\mathbf{C_i} \neq \mathbf{C_m}$).

For simplicity, it is assumed that the inhomogeneity and the matrix material are isotropic and Poisson's ratio of the inhomogeneity is the same as the matrix material. Then we have

$$\mathbf{C_i} = \alpha\mathbf{C_m}, \tag{3}$$

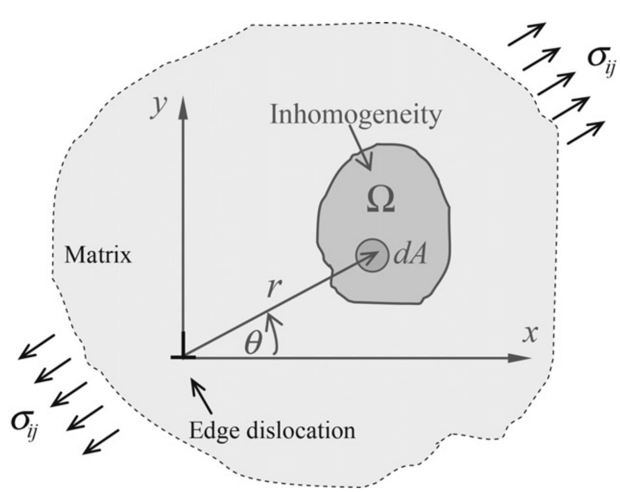

Fig. 1. An edge dislocation interacts with an inhomogeneity of arbitrary shape in an applied stress field.

where

$$\alpha = \frac{\mu_i}{\mu_m} \tag{4}$$

μ_i is the shear module of the inhomogeneity.

Combining Eqs. (1) and (3), it gives

$$\mathbf{e^T} = \mathbf{Le}, \tag{5}$$

where

$$\mathbf{L} = [(\alpha-1)\mathbf{S} + \mathbf{I}]^{-1}(1-\alpha), \tag{6}$$

$\mathbf{I}$ is the unit tensor. Thus, the tensor $\mathbf{L}$ relates the equivalent transformation strain $\mathbf{e^T}$ in the inhomogeneity to the combined strain $\mathbf{e}$ without going into the details of the form of the $\mathbf{C_i}$ and $\mathbf{C_m}$ tensors.

For a differential element with circular section inside the inhomogeneity, the non-zero components of the Eshelby tensor are given by Mura (1987)

$$\left.\begin{aligned} &S_{1111} = S_{2222} = \frac{5-4\nu}{8(1-\nu)}, \quad S_{1122} = S_{2211} = \frac{4\nu-1}{8(1-\nu)} \\ &S_{1133} = S_{2233} = \frac{\nu}{2(1-\nu)}, \quad S_{1212} = \frac{3-4\nu}{4(1-\nu)} \\ &S_{1313} = S_{2323} = \frac{1}{2} \end{aligned}\right\}. \tag{7}$$

Substituting Eq. (7) into Eq. (6) yields

$$\left.\begin{aligned} &L_{1111} = L_{2222} = \frac{(1-\alpha)(1-\nu)(3-4\nu+5\alpha-4\nu\alpha)}{(1+\alpha-2\nu)(1+3\alpha-4\nu\alpha)} \\ &L_{1122} = L_{2211} = -\frac{(1-\alpha)^2(1-\nu)(1-4\nu)}{(1+\alpha-2\nu)(1+3\alpha-4\nu\alpha)} \\ &L_{1133} = L_{2233} = \frac{(1-\alpha)^2\nu}{(1+\alpha-2\nu)}, \quad L_{3333} = (1-\alpha) \\ &L_{1212} = \frac{4(1-\alpha)(1-\nu)}{(1+3\alpha-4\nu\alpha)}, \quad L_{1313} = L_{2323} = \frac{2(1-\alpha)}{1+\alpha} \end{aligned}\right\}, \tag{8}$$

And other components of the $\mathbf{L}$ tensor are zero. Combining Eqs. (2), (5) and (8), the non-zero components of the transformation strain in dA are given by

$$\left.\begin{aligned} e_{11}^T &= L_{1111}e_{11} + L_{1122}e_{22} \\ e_{22}^T &= L_{2211}e_{11} + L_{2222}e_{22} \\ e_{12}^T &= L_{1212}e_{12} \end{aligned}\right\}, \tag{9}$$

for plane strain.

The elastic interaction energy per unit length in z-direction of dislocation on the differential element is given by Mura (1994)

$$dU_{\text{int}} = -\sigma_{ij}^d e_{ij}^T dA. \tag{10}$$

The non-zero components of σ_{ij}^d are

$$\left.\begin{aligned} \sigma_{11}^d &= \sigma_{22}^d = -\frac{\mu_m b}{2\pi r(1-\nu)}\sin\theta \\ \sigma_{12}^d &= \frac{\mu_m b}{2\pi r(1-\nu)}\cos\theta \end{aligned}\right\}. \tag{11}$$

Then we have

$$dU_{\text{int}} = \frac{C_1}{r}(2\sigma_{11}^d + \sigma_{11} + \sigma_{22})\sin\theta - \frac{C_2}{r}\left(\frac{1}{2}\sigma_{12}^d + \sigma_{12}\right)\cos\theta, \tag{12}$$

where

$$C_1 = \frac{b(1-\alpha)(1-2\nu)}{2\pi(1+\alpha-2\nu)}, \quad C_2 = \frac{2b(1-\alpha)}{\pi(1+3\alpha-4\nu\alpha)}, \tag{13}$$