Mechanics Research Communications 45 (2012) 77–84

Contents lists available at SciVerse ScienceDirect

Mechanics Research Communications

journal homepage: www.elsevier.com/locate/mechrescom

Pressure distribution in an elastomer confined by a long thin-walled flexible hollow cylinder under the assumption of a hydrostatic stress state

Frank Hofmann

Wehrwissenschaftliches Institut für Werk- und Betriebsstoffe (WIWeB), 85435 Erding, Germany

ARTICLE INFO

Article history:
Received 24 August 2011
Received in revised form 29 February 2012
Available online 24 April 2012

Keywords:
Confined elastomer
Compressed elastomer method
Shear lag
Hydrostatic stress state
Axisymmetry
Pressure distribution
Pressure loss
Flexible hollow cylinder
Bulk modulus

ABSTRACT

The stress distribution in a pressurized elastomer confined by a hollow cylinder is of interest in various applications of material testing and manufacturing. A relatively accurate closed form solution for the pressure distribution inside an elastomer confined by a rigid hollow cylinder was presented by Yu et al. (2001). But in many practical applications the assumption of a rigid hollow cylinder is not appropriate, because the cylinder deformations have a significant influence on the stresses inside the elastomer. Thus in this paper a solution for an elastomer confined by a deformable hollow cylinder is derived. Both axial and radial deformations of the hollow cylinder are taken into account, while the bending stiffness of the cylinder wall is neglected, i.e. the cylinder wall is treated according to the membrane theory. The accuracy of the proposed closed form solution is verified by a parametric finite element simulation.

1. Introduction

In order to measure the burst strength of tubes made of ceramic matrix composite (CMC) with possible applications as combustion chambers (Frieß et al., 2010) the compressed elastomer method described by Singh et al. (1996, 1997) and Carter (2006) was applied. This method uses a pressurized elastomer inside the cylindrical burst test specimen instead of a fluid. Compared to burst tests using a pressurized fluid the compressed elastomer method requires less effort, because seals and flanges are not necessary and no fluid spill occurs at specimen failure. A disadvantage of the compressed elastomer method is the axial load transfer between the elastomer and the hollow cylinder due to shear stress, which leads to a pressure loss along the cylinder axis. In order to assess this effect and to choose suitable dimensions and materials for the test setup an analytical model revealing the influence of different parameters on the pressure distribution was of interest. An exact analytical solution is not available, but an approximate closed form solution for the pressure inside an elastomer confined by a rigid hollow cylinder was presented in Yu et al. (2001) and a comparison with finite element simulations showed good agreement. But for the investigated CMC-tubes preliminary finite element simulations revealed that the deformations of the specimen may have distinct influence on the pressure distribution, thus an analytical model describing a flexible hollow cylinder was required. An estimation for the pressure distribution in an elastomer confined by flexible hollow cylinder is given in Singh et al. (1997), but this solution was not verified by numerical simulations and shows discrepancies from the result of Yu et al. (2001) if the rigid cylinder limiting case is considered. Therefore an effort was made to find an analytical model which can describe the pressure distribution in an elastomer confined by a deformable hollow cylinder and is consistent with the solution for the rigid cylinder limiting case presented by Yu et al. (2001).

2. Description of the investigated setup

The geometry, the boundary conditions and the coordinate system of the investigated setup are shown in Fig. 1a and b. A long thin-walled hollow cylinder made of a strong isotropic structural material, e.g. metal or ceramic, with Young's modulus E_{Cyl}, Poisson's ratio ν_{Cyl}, length L, inner radius R and wall thickness t_{Cyl} is considered. It is presumed that $L \gg R$ (long cylinder) and $t_{Cyl} < 0.1R$ (thin-walled hollow cylinder). The hollow cylinder is completely filled with a soft elastomer plug, which is characterized by its shear modulus G_{El} and its bulk modulus K_{El}. The outer surface of the elastomer plug is bonded to the hollow cylinder, i.e. no relative displacement is possible at the interface. In an experimental setup this may be achieved either by static friction or by adhesion.

E-mail addresses: frankhofmann@bundeswehr.org, FrankHofmann@bwb.org

0093-6413/$ – see front matter © 2012 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.mechrescom.2012.04.003

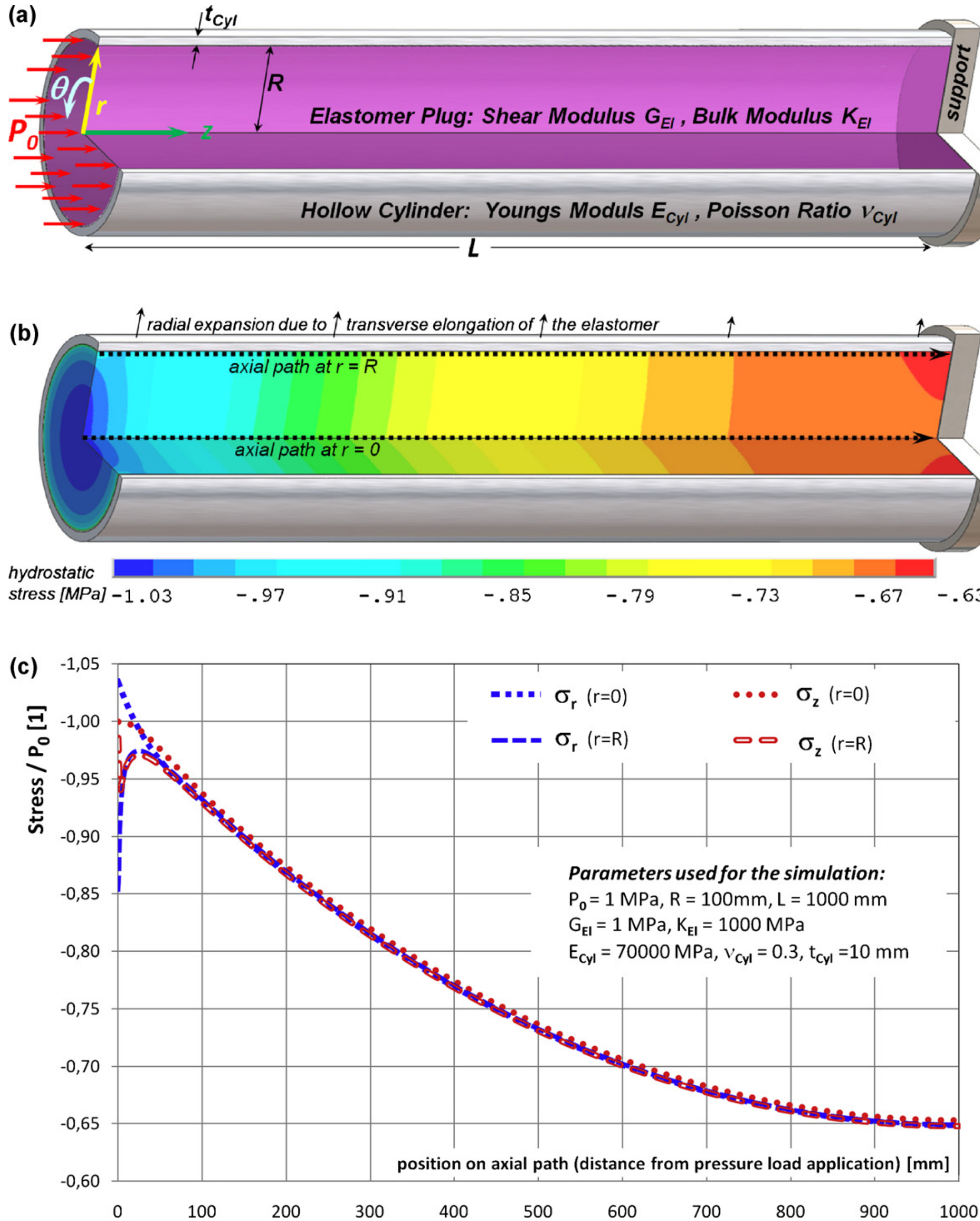

Fig. 1. Overview of the investigated setup. The geometry, the coordinate system, the pressure load P_0 and the support of the investigated setup are shown in (a). A contour plot of the hydrostatic stress inside the elastomer plug is shown in (b). The distribution of the stresses σ_r and σ_z in radial and axial direction along two paths (dotted lines in (b)) is shown in (c). Except for the region close to the pressure loaded end at $z=0$, the stresses on both paths are nearly equal. The parameters for the FE-simulation are indicated in (c).

On the right end of the setup (at axial position $z=L$), the hollow cylinder and the elastomer plug are frictionless supported, i.e. they can freely expand in radial r-direction, but no movement in the axial z-direction is allowed. This frictionless support is equivalent to a mirror symmetry boundary condition, thus the depicted setup can also be considered to be one half of a full setup which is mirror symmetric to the plane $z=L$.

On the left end (at $z=0$) the elastomer plug is loaded with an uniform pressure P_0 normal to the surface, while the end of the hollow cylinder at $z=0$ is free of axial loads. Thus the elastomer is under compressive stress and will react with a compressive volumetric strain according to its bulk modulus K_{El}. This causes a relative displacement between the hollow cylinder and the elastomer in axial direction, which is a function of the radial position r. The maximum relative axial displacement between elastomer and hollow cylinder is reached on the cylinder axis at $r=0$ and decays to zero from there to the inner surface of the hollow cylinder at $r=R$, where elastomer and hollow cylinder are bonded to each other (Fig. 2). Thus shear stresses occur, which cause a load transfer from the elastomer plug to the hollow cylinder. If a deformable hollow cylinder