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Free vibrations of three parameter functionally graded parabolic panels of revolution

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ABSTRACT

The aim of this paper is to deal with the dynamic behaviour of moderately thick functionally graded parabolic panels of revolution. A generalization of the power-law distribution presented in literature is proposed. The governing equations of motion are expressed in terms of five generalized displacement components of the points lying on the middle surface of the parabolic shell. The Generalized Differential Quadrature (GDQ) method is used to discretize the system equations. Numerical results concerning functionally graded parabolic panels show the influence of the three parameters of the power-law distribution on their mechanical behaviour.

1. Introduction

In this study, ceramic-metal graded shells of revolution with two different power-law variations of the volume fraction of the constituents in the thickness direction are considered. The effect of the power-law exponent and of the power-law distribution choice on the mechanical behaviour of functionally graded parabolic panels is investigated. In the last years, some researchers have analyzed various characteristics of functionally graded structures (Abrate, 2006; Bayat et al., 2008; Nie and Zhong, 2007; Patel et al., 2005; Pelletier and Vel, 2006; Zenkour, 2006), among others. However, this paper is motivated by the lack of studies in the technical literature concerning the free vibration analysis of functionally graded parabolic panels and the effect of the power-law distribution on their mechanical behaviour. A generalization of the power-law distribution available in literature is proposed. Two different three parameter power-law distributions are considered for the ceramic volume fraction. The homogeneous isotropic material can be inferred as a special case of functionally graded materials, too. From this point of view, the present work generalizes the paper by Tornabene and Viola (2008). A parametric study is undertaken, in order to give insight into the effect of the material composition on the natural frequencies of parabolic shell structures. Vibration characteristics are assessed by varying one parameter at a time of the power-law exponent distributions.

The present work is based on the First-order Shear Deformation Theory (FSDT) (Reddy, 2003). The geometric model refers to a moderately thick panel of revolution, and the effects of transverse shear deformation as well as rotary inertia are taken into account.

The numerical solution is obtained by using the Generalized Differential Quadrature (GDQ) method, which leads to a generalized eigenvalue problem, and is given in terms of generalized displacement components of the points lying on the middle surface of the shell panel. The main features of the numerical technique under discussion are illustrated in the book by Shu

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(2000). This simple and direct procedure can be applied to a large number of cases (Tornabene and Viola, 2007, 2008) to circumvent the difficulties of programming complex algorithms for the computer, as well as the excessive use of storage and computer time.

It should be noted that, in this paper we have extended the work by Tornabene and Viola (2009) by considering parabolic shell structures made of three parameter functionally graded materials. In all truth, this is the heart of the matter and the main novelty of the present paper. As a matter of fact, the classical volume fraction profiles used in the previous paper by authors can be obtained as special cases from the power-law distributions proposed in this paper.

2. Fundamental system for functionally graded panels of revolution

The basic configuration of the problem considered here is a doubly curved parabolic panel as shown in Fig. 1. The co-ordinates along the meridional and circumferential directions of the reference surface are φ and s, respectively. The distance of each point from the mid-surface of the shell panel along the normal is ζ. The panels considered are assumed to be single-layer shells of uniform thickness h. It should be noted that the geometry of the shell panels considered in this study is a part of a surface of revolution with a parabolic meridian.

The parabolic meridian can be described with the following equation:

$$(R_0 - R_b)^2 - kx_3' = 0 \tag{1}$$

where $k = (s_1^2 - d^2)/S$ is a characteristic parameter of the parabolic curve. The horizontal radius $R_0(\varphi)$ of a generic parallel of the shell represents the distance of each point from the axis of revolution x_3 and for a shell with parabolic meridian assumes the form:

$$R_0(\varphi) = \frac{k \tan\varphi}{2} + R_b \tag{2}$$

where R_b is the shift of the geometric axis of the meridian x_3' with reference to the axis of revolution x_3.

The radii of curvature $R_\varphi(\varphi), R_s(\varphi)$ in the meridional and circumferential directions and the first derivative of $R_\varphi(\varphi)$ with respect to φ can be expressed, respectively, as follows:

$$R_\varphi(\varphi) = \frac{k}{2\cos^3\varphi}, \quad \frac{dR_\varphi}{d\varphi} = \frac{3k\sin\varphi}{2\cos^2\varphi}, \quad R_s(\varphi) = \frac{k}{2\cos\varphi} + \frac{R_b}{\sin\varphi} \tag{3}$$

Finally, for doubly curved shells the Gauss-Codazzi relation assumes the form:

$$\frac{dR_0}{d\varphi} = R_\varphi \cos\varphi \tag{4}$$

Consistent with the assumptions of a moderately thick shell theory, the displacement field considered in this study is that of the First-order Shear Deformation Theory and can be put in the following form:

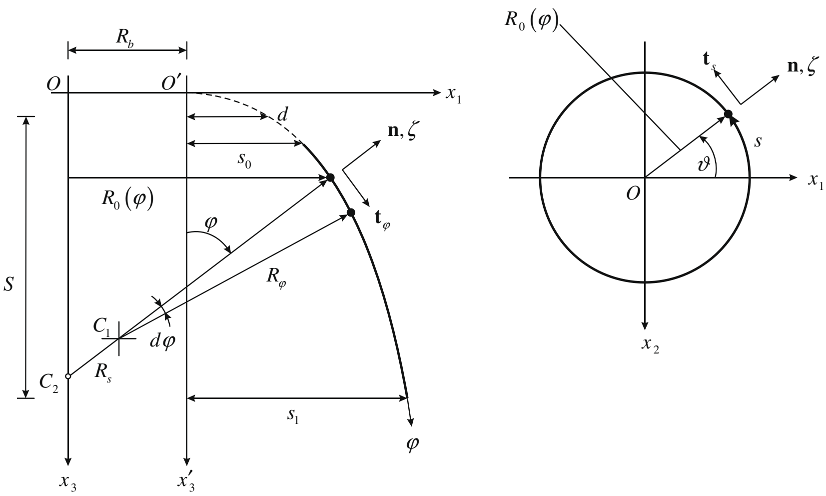

Fig. 1. Co-ordinate system and geometry of a doubly curved shell.