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Transient dynamic response of pile to vertical load in saturated soil

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ABSTRACT

This investigation is concerned with the dynamic response of a pile embedded in a saturated half-space subjected to transient vertical loading. The pile is represented by a beam element, and the saturated medium uses Biot's three-dimensional elastodynamic theory. The governing equation, which is formulated under the approximation that the strain of the fictitious pile is equal to the corresponding average over a circular area in the extended half-space, is found to be a Fredholm integral equation, and solved by an appropriate numerical method. Time domain solutions are obtained by using numerical Fourier inversion transformation. Some numerical results are given.

1. Introduction

Pile is often subjected to dynamic loads. Common examples are loads due to traffic, machinery, earthquakes and impact. The study of transient displacements and forces in a pile is important in the development of design for embedded foundations subjected to dynamic loads. Dynamic analysis of a pile has been mostly limited to the frequency domain due to the assumption of linearity in the various formulations that makes easier for the problem to be solved. Because of the analytical complexity involved, time domain analysis has been studied relatively less. The transient dynamic formulations are more complicated, and often pose a number of problems involving numerical accuracy and stability. The various approaches that have been used for transient problem can be categorized as continuum method, finite element method, boundary element method and dynamic Winkler method.

The early developments of the dynamic analysis of piles were based on the simplified continuum model developed by Baranov (1967) for embedded rigid foundations subjected to time-harmonic vertical loads. Nogami and Konagai (1986) presented an approximate time domain analysis to compute the response of an elastic pile subjected to a transient axial load. The governing equation of a pile is based on a finite difference approximation.

The boundary element approach is also used to study transient dynamic problem. A comprehensive review of the research pertinent to dynamic response of soil–structure interaction can be found in Cole et al. (1978), Manolis (1983), Karabalis and Beskos (1984), Banerjee et al. (1986), and Spyrakos and Beskos (1986). Mamoon and Banerjee (1992) proposed an approximated model to study the responses of piles under vertical and horizontal excitations. The soil domain was modeled by a hybrid BEM, while the pile was discretized by finite difference expressions. Lei et al. (1993) analyzed the transient vertical dynamic response of single piles in a layered half-space under a time-dependent vertical force by an FEM–BEM coupling approach. Cheung and Tham (1995) developed the general time domain boundary element in cylindrical co-ordinates for the study of dynamic response of single piles under horizontal transient excitations in a layered half-space. Tham et al. (1996)

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analyzed the transient response of vertical-loaded single piles by time-domain BEM. Militano and Rajapakse (1999) presented the dynamic response of an elastic pile subjected to transient torsional and axial loading in a multilayered soil. For a pile embedded in a multilayered soil, the impedance matrices of the pile segments were assembled by following the FEM. Time domain solutions were obtained by using numerical Fourier inversion procedure. Küçükarslan (2002) implemented a hybrid boundary element technique to model the time domain dynamic analysis of piles under impact loading.

Nogami and Konagai (1988) analyzed the dynamic response of pile foundations in the time domain using a Winkler approach. El Naggar and Novak (1996) presented a nonlinear analysis for pile groups in the time domain within the framework of Winkler hypothesis.

However, from the above–mentioned literatures the assumption that the medium is ideal elastic solid is not satisfactory for the cases of soil systems. For such cases, it may be more realistic to assume that the half-space is a two-phase medium. The first theory of propagation of wave in a fluid saturated porous medium was developed by Biot (1962). Biot's consolidation equations satisfy not only the requirements for the deformation of the solid matrix but also the Darcy's law for the fluid flowing in the medium. Zeng and Rajapakse (1999) extended the classical Muki and Sternberg (1970) formulation to analyze steady-state dynamic response of an axially loaded elastic bar partially embedded in a homogeneous poroelastic medium. Jin et al. (2001) studied the time-harmonic response of a pile in a poroelastic medium and under lateral loadings. Wang et al. (2003) investigated the dynamic response of pile groups embedded in a homogeneous poroelastic medium and subjected to vertical harmonic loading. Maeso et al. (2005) presented a three-dimensional boundary element model to analyze time-harmonic dynamic response of piles and pile groups in a saturated soil.

It is noted that the solution corresponding to the pile embedded in a saturated half-space subjected to transient loading is not reported in the literatures. The objective of this paper is to study the effect of pile under vertical transient loading in a saturated medium. The dynamic response of single pile in a homogeneous saturated medium uses Muki and Sternberg Method. The problem is decomposed into two systems: namely, an extended half-space and a fictitious pile with Young's modulus equal to the difference between Young's modulus of the real pile and the half-space. The load transfer problem is formulated in terms of a Fredholm integral equation of the second kind. Time domain solutions are obtained by using a numerical Fourier inversion transformation. Some numerical solutions are given in this paper.

2. Governing equations

Consider the axisymmetric response of a saturated medium. According to the consolidation theory of Biot for a completely saturated medium, the governing equations can be expressed in terms of the displacements and the pore pressure as follows (Biot, 1962):

$$G\left(\nabla^2-\frac{1}{r^2}\right)u_r+(\lambda+G)\frac{\partial e}{\partial r}-\frac{\partial p_f}{\partial r}=\rho\ddot{u}_r \tag{1a}$$

$$G\nabla^2u_z+(\lambda+G)\frac{\partial e}{\partial r}-\frac{\partial p_f}{\partial z}=\rho\ddot{u}_z \tag{1b}$$

$$-\frac{\partial p_f}{\partial r}=\frac{1}{k_d'}\dot{w}_r+\rho_f\ddot{u}_r \tag{1c}$$

$$-\frac{\partial p_f}{\partial z}=\frac{1}{k_d'}\dot{w}_z+\rho_f\ddot{u}_z \tag{1d}$$

where u_r and u_z are the displacements of the solid matrix in the r and z directions, respectively; w_r and w_z are the fluid displacements relative to the solid matrix in the r and z directions, respectively; p_f is the pore pressure; ρ, ρ_s and ρ_f are the mass densities of the bulk material, the soil and the pore fluid, respectively, $\rho=(1-n)\rho_s+n\rho_f$; n is the porous ratio; λ and G are the Lame constants; k_d' is the dynamic permeability; ∇^2 is the Laplace operator; e is the dilatation; overdots denote the derivatives of field variables with respect to time t.

The constitutive relations of a homogenous saturated material can be expressed as

$$\sigma_{zz}=\lambda e+2G\frac{\partial u_z}{\partial z}-p_f,\quad \sigma_{zr}=G\left(\frac{\partial u_r}{\partial z}+\frac{\partial u_z}{\partial r}\right),\quad q_z=\dot{w}_z \tag{2a-c}$$

where q_z is the fluid discharge.

Consider the soil and the fluid are incompressible, the continuity equation is

$$\frac{\partial\dot{u}_r}{\partial r}+\frac{\dot{u}_r}{r}+\frac{\partial\dot{u}_z}{\partial z}+\frac{\partial\dot{w}_r}{\partial r}+\frac{\dot{w}_r}{r}+\frac{\partial\dot{w}_z}{\partial z}=0 \tag{3}$$

Differentiation of Eqs. (1a) and (1b), $\frac{\partial(1a)}{\partial r}+\frac{(1a)}{r}+\frac{\partial(1b)}{\partial z}$, and differentiation of Eqs. (1c) and (1d), $\frac{\partial(1c)}{\partial r}+\frac{(1c)}{r}+\frac{\partial(1d)}{\partial z}$ yield

$$\nabla^2p_f=\frac{1}{k_d'}\dot{e}-\rho_f\ddot{e},\quad \nabla^2p_f=M\nabla^2e-\rho\ddot{e} \tag{4a, b}$$

where $M=\lambda+2G$