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Strict Lyapunov function for sliding mode control of manipulator using quasi-velocities

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This paper describes proposition of sliding mode control using strict Lyapunov function for robot manipulators described in terms of some quasi-velocities (QV). The considered here quasi-velocities contain both kinematical and dynamical parameters of a manipulator. Introducing QV together with joint position leads to one first-order dynamic equations with diagonal mass matrix instead of a second-order differential equation and one first-order equation which describes transformation between joint velocities and QV. The presented controller based on strict Lyapunov function guarantees closed-loop stability and global positioning. Differences between classical sliding mode control scheme and the considered here were shown on 3 *d.o.f.*, 3-*D* Yasukawa-like robot.

1. Introduction

Trajectory control problem arises if the manipulator is required to follow a desired trajectory. In the robotic literature two approaches are usually used: computer torque (inverse dynamic control) and sliding mode control (Sciavicco and Siciliano, 1996; Slotine and Li, 1991; Spong, 1989; Wen, 1990; Wen and Bayard, 1988). Sliding mode approach (Slotine and Li, 1987; Slotine and Li, 1991) relies on exploitation the structure of Lagrangian formulation for rigid manipulators without linearization its dynamic equations. For example a control scheme combining the saturation and integral control is described in Liu and Goldenberg (1996). Also Spong (1992) presented a modified controller based on strict Lyapunov function. One of applications allows to control a shape (Mochiyama et al., 1999). Sliding mode control is also useful for flexible manipulators (Arteaga, 2003).

Classical description leads to obtaining second-order nonlinear differential equations of motion. These equations involve both generalized position vector and velocity vector which represent a joint space of manipulator. However for control purposes first-order equations of motion with diagonal mass matrix seem more convenient than the second-order equations. There exist several methods which result in such equations. All of them are based on some quasi-velocities which depend on the kinematic and dynamic parameters of the manipulator (e.g. Jain and Rodriguez, 1995; Loduha and Ravani, 1995). Jain and Rodriguez (1995) presented diagonalized dynamics for manipulators which used normalized (NQV) and unnormalized (UQV) quasi-velocities. The obtained equations of motion was based on the spatial operator algebra. On the contrary Loduha and Ravani proposed in Loduha and Ravani (1995) a rate transformation matrix, which served for decoupling of the dynamical equations of motion. The obtained matrix was congruent to the original system mass matrix. This method was related to the modified Kane's equations given e.g. in Kane and Levinson (1983).

In this paper we propose a sliding mode controller in terms of UQV or GVC using a strict Lyapunov function. By the strict Lyapunov function we mean a radially unbounded and globally positive definite function whose time derivative along all

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trajectories of a closed-loop system results in a globally negative definite function. A class of such functions (but different as the given here) was considered in Santibanez and Kelly (1997). The strict Lyapunov function for classical equations of motion was given by Spong Spong, 1992. The second aim is to point at some advantages which offers the control scheme in terms of QV. It is also shown features which are observable if the system under the new control is considered. They arise from the fact that QV are decoupled in the kinetic energy sense and lead to decoupling of the mass matrix of the manipulator.

The paper is organized as follows. Section 2 gives diagonalized equations of motion in terms of QV. In Section 3 the sliding mode controller in joint space of manipulator is presented. Simulation results comparing performance between the new control scheme and the classical controller for 3 *d.o.f.*, 3-*D* Yasukawa-like robot are contained in Section 4. The last section offers conclusions.

2. Dynamics in terms of quasi-velocities

Recall that the classical equations of motion for a manipulator can be written in the following form Slotine and Li (1987, 1991):

$$M(\theta)\ddot{\theta} + C(\theta,\dot{\theta})\dot{\theta} + G(\theta) = \tau, \tag{1}$$

where $\mathcal{N}$ is the number of degrees of freedom, $\theta, \dot{\theta}, \ddot{\theta} \in R^{\mathcal{N}}$ is the vectors of generalized positions, velocities, and accelerations, respectively, $M(\theta) \in R^{\mathcal{N}\times\mathcal{N}}$ is the system mass matrix, $C(\theta,\dot{\theta}) \in R^{\mathcal{N}}$ is the matrix of Coriolis and centrifugal forces in classical equations of motion, $G(\theta) \in R^{\mathcal{N}}$ is the vector of gravitational forces in classical equations of motion, $\tau \in R^{\mathcal{N}}$ is the vector of generalized forces.

Assuming that there exist some positive constants β_m, β_M, β_c, β_g, and vector x the following properties can be established (Arteaga, 2003; Sciavicco and Siciliano, 1996; Slotine and Li, 1991; Spong, 1992) (I denotes the identity matrix):

(P1) The inertia matrix $M(\theta)$ satisfies the inequality $\beta_m I \leqslant M(\theta) \leqslant \beta_M I,\ \forall\theta \in R^{\mathcal{N}}$.
(P2) Matrix $C(\theta,\dot{\theta})$ satisfies $C(\theta,\dot{\theta}) \leqslant \beta_c\|\dot{\theta}\|$, $\forall\dot{\theta} \in R^{\mathcal{N}}$.
(P3) One can define skew symmetric matrix: $x^{\mathrm{T}}[\frac{1}{2}\dot{M}(\theta) - C(\theta,\dot{\theta})]x = 0$, $\forall x \in R^{\mathcal{N}}$.
(P4) The gravity vector $G(\theta)$ is bounded as $\|G(\theta)\| \leqslant \beta_g$, $\forall\theta \in R^{\mathcal{N}}$.

Based on the method offered in Loduha and Ravani (1995) (for GVC) the same manipulator we can describe with two first-order equations: the diagonalized equation of motion and the velocity transformation equation:

$$H\dot{\zeta} + C(\theta,\zeta)\zeta + G_\zeta(\theta) = \rho \tag{2}$$

$$\dot{\theta} = B\zeta \tag{3}$$

where matrices and vectors are given as follows ($\dot{B}$ denotes time derivative of B):

$$H = B^{\mathrm{T}}M(\theta)B \tag{4}$$

$$C(\theta,\zeta) = B^{\mathrm{T}}[M(\theta)\dot{B} + C(\theta,\dot{\theta})B] \tag{5}$$

$$G_\zeta(\theta) = B^{\mathrm{T}}G(\theta)), \rho = B^{\mathrm{T}}\tau. \tag{6}$$

In Eqs. (2)–(6) H is a diagonal matrix congruent to mass matrix of manipulator $M(\theta)$ (this matrix can be obtained using the method described in Loduha and Ravani (1995)), $\zeta, \dot{\zeta}$ are vectors of inertial quasi-velocities and its time derivative, respectively, $C(\theta,\zeta)$ is a new Coriolis force vector, $G_\zeta(\theta)$ is a new gravitational forces vector, and ρ is a vector of quasi-forces. The invertible matrix B is the rate transformation matrix which converts joint velocities into generalized velocity components.

Remark 1. It is assumed that the properties (P1)–(P4) are true. The mass matrix B arises from decomposition of the matrix $M(\theta)$ then from (P1) boundedness of the matrix H is guaranteed. From (P2)–(P3) one can conclude that $C(\theta,\zeta)$ is bounded (because the matrix $\dot{B}$ results from $\dot{M}(\theta)$). Also vector $B^{\mathrm{T}}G(\theta)$ is bounded because of the property (P1) and (P4). If we use UQV (Jain and Rodriguez, 1995) instead of GVC Eqs. (2)–(6) are also valid in this general form.

Remark 2. From Eqs. (3) and (4) arises that the kinetic energy of the manipulator can be expressed as follows:

$$\mathcal{K}(\theta,\zeta) = \frac{1}{2}\dot{\theta}^{\mathrm{T}}M(\theta)\dot{\theta} = \frac{1}{2}\zeta^{\mathrm{T}}B^{\mathrm{T}}M(\theta)B\zeta = \frac{1}{2}\zeta^{\mathrm{T}}H\zeta. \tag{7}$$

This expression says that each quasi-velocity ζ_k can be considered separately in the sense of the kinetic energy. It is because a part of the kinetic energy of each link which concerns internal couplings is transformed into k-th quasi-velocity.

Remark 3. The matrix H is diagonal and hence each k-th equation of motion is decoupled from other equations. The diagonal elements of this matrix allows one to detect the total inertia which is transferred by each joint. The total inertia depends on the manipulator design and on the used decomposition method (i.e. UQV or GVC). However in each case it represents some inertia arising from the fact that the manipulator is not rigid body but a multi-rigid body.