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Dynamic equilibrium equations of linear piezoelectric Euler–Bernoulli beams

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ABSTRACT

A one-dimensional model for the dynamics of linear piezoelectric straight prismatic beam based on the Euler–Bernoulli's theory and appropriate hypotheses on the electric displacement field is developed. The equations of motion of longitudinal and flexural vibrations are formulated in terms of one-dimensional mechanical and electrical displacement fields. The formulation uses the beam equilibrium equations including inertia forces and one-dimensional versions of Gauss–Maxwell equations. Applied approach yields a technical theory of linear piezoelectric beams, it rests on ad hoc assumptions in order to neglect certain terms, the mechanical displacements and axial component of the electric displacement are assumed to be linear functions of the cross-sectional coordinates.

1. Introduction

Since the brothers Curie discovered the piezoelectricity in 1880 many scientists have investigated extensively the piezoelectric materials which are widely used to design smart structures. This statement is documented by many articles on the subject (Daví, 1996; Dökmeci, 1988; Loewy, 1967; Rao and Sunar, 1994; Smith, 1992; Sun et al., 2001) as well as by some dedicated books such as Cady (1964), Manson (1950), Tzou (1993), Yang (2005, 2006). A lot of papers deal with the dynamics of piezoelectric beams (Daví, 1997; Dökmeci, 1974; Paul and Natarajan, 1994; Wang and Quek, 2000). Extensional and flexural motions with shear deformations of electroelastic beams with rectangular cross section are derived from the three-dimensional equations using double power series expansions defined for the thickness and width directions by Yang (1998, 2005, 2006); Yang and Zhang (1999); and Yang et al. (1999, 2000). In the above mentioned papers the electric field is applied in thickness direction and the thickness and axially poling ceramic beams are considered. In the most cases upper and lower faces of ceramic beams with rectangular cross-section are electroded. The papers of Yang (1998, 2005, 2006); Yang and Zhang (1999); and Yang et al. (1999, 2000) focus on dynamic theories of piezoelectric structures for device applications. Theoretical analysis for electroelastic beams of generic cross-section is presented by Daví (1997) and Dökmeci (1974). Paper by Daví (1997) presents a one-dimensional model for the dynamics of linear piezoelectric rods starting from the equations of the three-dimensional linear piezoelectricity. The method of internal constraint and the concept of electrical thinness are used by Daví (1997). Equations of motion with boundary conditions for extensional, flexural and torsional vibrations are obtained with the help of a variational theorem deduced from Hamilton's principle (Daví, 1997). Dökmeci (1974) presented a higher order linear theory of piezoelectric crystal bars, in which, a power series representation in cross-sectional coordinates is employed for both the mechanical displacement and electric potential fields. The governing equations of the piezoelectric

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crystal bars, by the use of power series representation, are deduced from a variational theorem. A set of one-dimensional approximate equations of motion, charge equations of electrostatics, strain–displacement and electric field – electric potential relations and constitutive equations with the initial and boundary conditions are derived by Dökmeci (1974). The character of the present paper is mainly theoretical. The aim is to derive the equations of motion of axially polarized linear piezoelectric beam for longitudinal and flexural vibrations. The assumed form of the mechanical displacements satisfies the requirements of the Euler–Bernoulli beam model, that is, the cross-section is rigid in its plane and remains orthogonal to the deformed axis. It is assumed that the axial component of the electric displacement is a linear function of the cross-sectional coordinates. This assumption is more general as used in paper by Daví (1997), where it has been assumed the axial component of the electric displacement is constant on the cross-section. In this paper, a direct method, which does not use the variational formulation, is presented to get a technical theory for linear piezoelectric beams. The technical theory of beams may be characterized as approximation for linear theories in which the displacement varies linearly in the cross-sectional coordinates, in our case, the displacement is subjected to the Euler–Bernoulli's hypothesis. The effects of the transverse shear deformations are neglected. In cases of technical interest, the technical theory yields good results for displacements and stress resultants (Daví, 1992).

2. Derivation of expressions of normal stress and electric potential

Let $B = A \times (0,L)$ be a right cylinder of length L, with its cross-section A which may be a simply or multiply-connected bounded regular region of R^2. Let A_1 and A_2 be the bases, and $A_m = \partial A \times (0,L)$ the mantle of B. The boundary surface of B consists of three part as, $\partial B = A_1 \cup A_2 \cup A_m$. The rectangular Cartesian coordinate frame $Ox_1x_2x_3$ is supposed to be chosen in such a way that axis Ox_3 is parallel to the generators of the cylindrical boundary surface segment A_m and the plane Ox_1x_2 contains the terminal cross-section A_1. The position of end cross section A_2 is given by $x_3 = L$. A point P in $\bar{B} = B \cup \partial B$ is indicated by the vector $\vec{OP} = \boldsymbol{r} = x_1\boldsymbol{e}_1 + x_2\boldsymbol{e}_2 + x_3\boldsymbol{e}_3 = \boldsymbol{R} + x_3\boldsymbol{e}_3$, where $\boldsymbol{e}_1$, $\boldsymbol{e}_2$ and $\boldsymbol{e}_3$ are the unit vectors of the coordinate system $Ox_1x_2x_3$ (Fig. 1). Denote $\boldsymbol{u} = \boldsymbol{u}(x_1,x_2,x_3,t)$ the displacement field, where t is the time $(0 \leqslant t \leqslant \tau)$. The longitudinal and flexural motions of the considered beam is approximated by the next displacement field according to the Euler–Bernoulli's hypothesis

$$\boldsymbol{u}(x_1,x_2,x_3,t) = \boldsymbol{U}(x_3,t) + \left(u(x_3,t) - \frac{\partial \boldsymbol{U}}{\partial x_3}\cdot \boldsymbol{R}\right)\boldsymbol{e}_3, \tag{1}$$

$$\boldsymbol{U} = \boldsymbol{U}(x_3,t) = U_1(x_3,t)\boldsymbol{e}_1 + U_2(x_3,t)\boldsymbol{e}_2. \tag{2}$$

In Eq. (1), the scalar product of two vectors is denoted by dot (Lurje, 1970; Malvern, 1969). The strain–displacement relationships of the linearized theory of elasticity give for the strains (Lurje, 1970; Malvern, 1969)

$$\varepsilon_{11} = \varepsilon_{22} = \varepsilon_{12} = \varepsilon_{13} = \varepsilon_{23} = 0, \tag{3}$$

$$\varepsilon_{33} = \frac{\partial u}{\partial x_3} - \frac{\partial^2 \boldsymbol{U}}{\partial x_3^2}\cdot \boldsymbol{R}. \tag{4}$$

Let $\boldsymbol{D} = \boldsymbol{D}(\boldsymbol{R},x_3,t)$ be the electric displacement field which has the next representation,

$$\boldsymbol{D} = D_1(\boldsymbol{R},x_3,t)\boldsymbol{e}_1 + D_2(\boldsymbol{R},x_3,t)\boldsymbol{e}_2 + D_3(\boldsymbol{R},x_3,t)\boldsymbol{e}_3 = \boldsymbol{d}(\boldsymbol{R},x_3,t) + D_3(\boldsymbol{R},x_3,t)\boldsymbol{e}_3. \tag{5}$$

It is assumed that the axial component of the electric displacement field is a linear function of the cross-sectional coordinates x_1, x_2, that is

$$D_3(\boldsymbol{R},x_3,t) = \frac{\partial d_0}{\partial x_3} + \frac{\partial \boldsymbol{d}_1}{\partial x_3}\cdot \boldsymbol{R}, \quad d_0 = d_0(x_3,t), \quad \boldsymbol{d}_1 = \boldsymbol{d}_1(x_3,t) \quad \boldsymbol{d}_1\cdot\boldsymbol{e}_3 = 0. \tag{6}$$

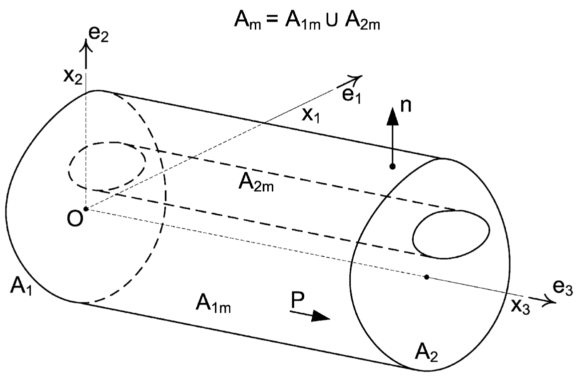

Fig. 1. Geometry of axially polarized linear piezoelectric beam.