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A note on a reappraisal and generalization of the Kelvin–Voigt model

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ABSTRACT

In this short note we show that expressing the strain and the symmetric part of the velocity gradient as functions of the stress instead of providing constitutive relations for the stress, as is the norm, allows one to obtain a far wider class of models to describe the response of viscoelastic bodies than those that are possible within the classical framework. Such an approach is used to describe the Kelvin–Voigt solid and several of its generalizations. A further generalization that recognizes that a certain class of viscoelastic solids, such as the Kelvin–Voigt solid, can be thought of a mixture of a dissipative fluid and an elastic solid, allows one to develop models wherein implicit relationships can be provided for the dissipative response and the elastic response.

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1. Introduction

In this short note we would like to provide an alternate way of looking at, and expressing, the constitutive relation for the classical Kelvin–Voigt model (see Thomson, 1865, Voigt, 1892) that lends itself to generalizations that are in keeping with a fundamental departure from the classical approach for the development of constitutive relations, namely that of expressing the deformation as a function of the stress, or in more general instance the history of the stress, or in an even more general setting providing a "relation" between the history of kinematical quantities and the history of the stress. It is very clear that force/stress cause[1] motion/deformation. If we are to assume that the stress is the cause and the deformation the effect, it would make eminent sense to express the effect as a function of the cause (and not the cause as a function of the effect which is what one does when expressing the stress as a function of the deformation), or if this is not possible to provide a relationship between the stress (its history) and the deformation (its history). The current wisdom of providing constitutive relations for the stress in terms of kinematical quantities is putting the cart before the horse and seems to stem from mathematical considerations rather than physical or philosophical reasons. Expressing the stress as a function of the displacement gradient, as is the case in linearized elasticity, or as a function of the velocity gradient as is the case in classical fluid mechanics allows one to reduce the problem to solving a partial differential equation for the displacement and velocity gradient, respectively. On the other hand, providing general "relationships" between the appropriate kinematical quantities and the stress will lead to a very large system of equations that need to be solved simultaneously, as will become clear in this article. The mathematical development of the times was incapable of handling such complex systems of equations and hence the reason for resorting to what seems an inappropriate manner for describing the response of bodies (see Rajagopal, submitted for publication, for a detailed discussion of the relevant issues). While in classical linearized elasticity (linearized viscoelasticity) one can express either the stress or strain in terms of the other quantity (or its history in the case of linear viscoelasticity), in general non-linear theories of elasticity the stress is expressed as a function of the non-linear strain (in non-linear theories for viscoelastic bodies the stress is expressed

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[1] We shall not get into a philosophical discussion concerning causality or the relationship between cause and effect in this short article.

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in terms of the history of the deformation as in the case of the K-BKZ model (see Bernstein et al., 1993), though implicit relation are also commonly used for viscoelastic fluids (e.g., the Oldroyd-B fluid Oldroyd, 1950 or Burgers' fluid Burgers, 1939).

The classical one-dimensional Kelvin–Voigt model can be thought of as a mixture of a linearized elastic solid and a linearly viscous fluid that co-exist with no relative motion between the solid and the fluid, and the mechanical analog of a spring and dashpot in parallel reflects this fact. A generalization of such a mixture would be the co-existence of a general elastic solid and a general viscous fluid. Recently, it has been shown (see Rajagopal, 2007, Rajagopal and Srinivasa, 2007) that the notion of an elastic body, if by that we mean bodies that are incapable of dissipation, can be greatly generalized to include models that bear an implicit relations between the stress and the strain. Similarly, viscous fluids can also be generalized to those given by implicit constitutive relations between the stress and the symmetric part of the velocity gradient as well as more general implicit relations (see Rajagopal, 2006). Thus, it would be possible to generalize the classical Kelvin–Voigt solid as a mixture of more general elastic solids and viscous fluids. Such a mixture presents several interesting features. As one cannot substitute for the stress in terms of the strain or the strain rate, as we deal with implicit constitutive relations, we need to solve the constitutive relations and the balance equations for mass, and linear momentum, simultaneously. This leads to a system of partial differential equations of which the constitutive relations are a part of the system, unlike the system that one obtains when one substitutes the explicit expression for the stress in the balance of linear momentum, in the classical case.

2. The non-linear Kelvin–Voigt model and some generalizations

The mechanical analog for the one-dimensional Kelvin-Voigt model consists in a spring and dashpot in parallel. If σ denotes the applied stress and σ_s and σ_d the stresses supported by the spring and dashpot, and if ε is the strain, then (see Wineman and Rajagopal, 2001)

$$\sigma_s = \kappa\varepsilon, \sigma_d = \eta\dot{\varepsilon}, \sigma_s + \sigma_d = \sigma, \tag{1$_{1-3}$}$$

where κ and η are the spring constant and the viscosity, respectively. This immediately leads to the one-dimensional Kelvin–Voigt model

$$\sigma = \kappa\varepsilon + \eta\dot{\varepsilon}, \tag{2}$$

The balance of linear momentum, for the one-dimensional problem is

$$\frac{\partial\sigma}{\partial x} = \rho\frac{\partial^2 u}{\partial t^2}, \tag{3}$$

where u is the displacement. Since the strain is given by

$$\varepsilon = \frac{\partial u}{\partial x}, \tag{4}$$

we have two unknowns u and σ, and two Eqs. (2) and (3).

The three dimensional generalization for the case of an incompressible viscoelastic solid takes the form

$$\mathbf{T} = -p\mathbf{1} + \mu\mathbf{B} + \eta\mathbf{D}, \tag{5}$$

where μ is the shear modulus, η is twice the viscosity, $-p\mathbf{1}$ is the reaction stress due to the constraint of incompressibility, and

$$\mathbf{B} = \mathbf{F}\mathbf{F}^{\mathrm{T}}, \quad \mathbf{F} = \mathbf{1} + \frac{\partial\mathbf{u}}{\partial\mathbf{x}}, \tag{6$_{1-2}$}$$

$$\mathbf{D} = \frac{1}{2}[\mathbf{L} + \mathbf{L}^{\mathrm{T}}], \quad \mathbf{L} = \dot{\mathbf{F}}\mathbf{F}^{-1} = \frac{\partial\mathbf{v}}{\partial\mathbf{x}}, \tag{7$_{1-2}$}$$

$\mathbf{u}$ is the displacement and $\mathbf{v}$ the velocity. Since the body is incompressible, it can only undergo isochoric motion and hence has to meet the constraint

$$\det\mathbf{F} = 1. \tag{8}$$

Now the unknowns are the displacement field $\mathbf{u}$ and the pressure p, namely four scalar unknowns, while we have the balance of linear momentum and the constraint (8), four scalar equations.

We can also consider the compressible version of the Kelvin–Voigt model, namely

$$\mathbf{T} = \mu\mathbf{B} + \eta\mathbf{D}, \tag{9}$$

and in this case we need to satisfy the balance of mass

$$\rho(\det\mathbf{F}) = \rho_0, \tag{10}$$

as well as the balance of linear momentum

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