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Mechanics of wrinkle/ridge transitions in thin film/substrate systems

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ABSTRACT

The mechanics of the formation and propagation of ridges on compressed stiff film/compliant substrate systems is studied theoretically and experimentally. Ridges form on bilayer systems where the elastomeric substrate is subject to a significant pre-stretch prior to attachment of the film. When the bilayer is then subject to increasing overall compressive strain, sinusoidal wrinkles first form and subsequently become unstable giving way to localized ridges with relatively large amplitudes. Two-dimensional plane strain simulations for neo-Hookean film/substrate systems reveal the transition from wrinkles to ridges under increasing compression and the reverse transition from ridges to wrinkles when the overall compression is subsequently reduced. For a significant range of pre-stretch, the two transition strains differ, and a significant hysteresis response is observed in a complete cycle of loading and unloading. The Maxwell equal-energy condition has been identified associated with co-existence of wrinkles and ridges and with the three-dimensional steady-state propagation condition for the ridges. Experiments conducted with a specially designed film/substrate loading system have been performed that confirm the essential features of ridge formation and the hysteretic behavior in loading/unloading cycles that span the two transitions.

1. Introduction

When a stiff thin film supported by a compliant thick substrate is compressed to a critical strain, the flat surface loses stability and forms periodic wrinkles (Bowden et al., 1998; Nowinski, 1969). In recent years the notion that wrinkles are a failure mode which should be suppressed has been replaced by efforts to make creative use of wrinkles. Published efforts have included the fabrication of stretchable electronics (Khang et al., 2006), measurement of mechanical properties (Stafford et al., 2004), assembly of particles (Lu et al., 2007; Schweikart and Fery, 2009), changing optical properties (Kim et al., 2013; Lee et al., 2010), and tuning surface adhesion and wettability (Chan et al., 2008; Chung et al., 2007; Lin et al., 2008; Lin and Yang, 2009). The reversibility of the elastic deformation associated with wrinkling in these applications allows the systems to be repeatedly cycled between flat and wrinkle states. Various wrinkle morphologies have also been studied, such as sinusoidal, herringbone, checkerboard and hexagonal modes (Audoly and Boudaoud, 2008; Bowden et al., 1998; Cai et al., 2011), induced by varying the loading condition, such as from uniaxial to biaxial compression (Breid and Crosby, 2011), and

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controlling the deterministic order of wrinkles (Lin and Yang, 2007; Yin et al., 2012).

After the formation of wrinkles, if the compressive strain is further increased, new advanced modes can appear such as wrinkles that double their periodicity and ultimately form localized folds (Brau et al., 2013, 2011; Holmes and Crosby, 2010; Kim et al., 2011; Pocivavsek et al., 2008; Sun et al., 2012). Only recently has it become apparent, that post-wrinkling bifurcations depend in a significant way on pre-stretch of the substrate (Auguste et al., 2014; Sun et al., 2012). If the substrate is subject to a tensile pre-stretch, wrinkles are stabilized and period-doubling does not usually occur until overall compressive strains of about 20% are imposed if the film/substrate modulus ratio is large (Auguste et al., 2014; Cao and Hutchinson, 2012; Chen and Crosby, 2014). The stabilized wrinkles grow to a relatively high aspect ratio (Chen and Crosby, 2014). If the substrate is subject to a mild pre-compression, wrinkles are destabilized and period-doubling occurs earlier at a lower overall compressive strain (Auguste et al., 2014). If the substrate is subject to a large pre-compression of around 0.3 strain, the wrinkle period appears to multiply chaotically (Auguste et al., 2014).

It has also been discovered that if the substrate is subject to a sufficiently large pre-tension prior to film attachment, a different mode of post-wrinkling can occur termed the localized ridge mode (Cao et al., 2014; Cao and Hutchinson, 2012; Takei et al., 2014; Wang and Zhao, 2013; Zang et al., 2012). In the two-dimensional context, one out of perhaps five or ten wrinkles grows to a large amplitude forming a ridge with the remaining wrinkles reduced to an almost flat state. Although folds and ridges are both localized, their morphologies are very different. A fold bends the film into a tight loop that pushes into the substrate, while a ridge is an open bend which grows outwards from the substrate pulling the substrate with it. These differing post-wrinkling behaviors can be explained qualitatively by the highly nonlinear elasticity of the elastomeric substrate. A significant pre-tension of the substrate makes it easier to pull material outward from the surface than to push material into the surface, while a pre-compression has the opposite effect (Cao and Hutchinson, 2012; Zang et al., 2012). The ridge instability mode was first noted in a numerical simulation (Cao and Hutchinson, 2012), and then shortly thereafter observed in experiments (Cao et al., 2014; Chen and Crosby, 2014; Takei et al., 2014; Wang and Zhao, 2013; Zang et al., 2012). The large aspect ratio of the ridge instability facilitates its applications in reversible wettability tuning and applications involving cell alignment (Cao et al., 2014).

It has been shown within a two-dimensional context that the wrinkle to ridge transition is unstable – that wrinkles will snap dynamically to ridges (Takei et al., 2014). However, the detailed mechanics of the formation and propagation of ridges has not been studied. In particular, the implications of the distinct wrinkle-to-ridge and ridge-to-wrinkle transitions and the associated hysteretic behavior under cycles of compression remain unclear. In this paper, we use the finite element method to study the formation, propagation, hysteresis and geometry of ridges. A detailed exploration of the two-dimensional behavior of ridges is given using plane strain simulations as depicted in Fig. 1a. These provide conditions for the transitions from wrinkles to ridges and vice versa, together with hysteric effects associated with cycles of compression which span the transitions. Based on arguments associated with the Maxwell condition for co-existing phases, these same two-dimensional simulations can be used to derive conditions governing the co-existence of wrinkles and ridges and for the three-dimensional steady-state propagation of the ridges, as depicted in Fig. 1b. The present study is purely mechanical and conducted within the framework of nonlinear continuum elasticity. The wrinkles and ridges can be interpreted as distinct phases and, as such, the system represents a mechanical realization of phase transitions within the larger setting of material phase transitions (Porter and Easterling, 1981; Balluffi et al., 2005). Parallels exist between the present system and martensitic phase transitions studied extensively in the materials and mechanics literature (e.g., Zhang et al., 2009). To simulate the wrinkle to ridge transition and its reverse, we have exploited two numerical techniques which will be described in the body of the paper: the static force–displacement method and pseudo-dynamic loading–unloading method.

Full details of the simulations for one particular case are presented in Section 2 introducing the definitions of the two transformation strains, the Maxwell strain and the hysteresis behavior under cyclic overall compression. The sensitivity of the computational model to some of the modeling assumptions is presented in Section 3 along with simulations which reveal the role of substrate pre-stretch and the redistribution of the energy in the wrinkle to ridge transition. Experimental results demonstrating ridge formation and cyclic loading hysteresis for a specific stiff film/substrate bilayer are obtained using a specially designed pre-stretching/compression system. These are presented in Section 4. Concluding remarks are given in Section 5.

2. Analysis of ridge formation, propagation and critical transitions

The formation and propagation of ridges in a film/substrate bilayer with both materials modeled as neo-Hookean is investigated in this section. As noted in the Introduction, a substantial pre-stretch of the substrate is required for ridges to form. In the simulations in this paper, both the pre-stretch and the subsequent overall compression of the bilayer are taken to be plane strain deformations. Two-dimensional wrinkle and ridge patterns with no out-of-plane variation are analyzed in detail using plane strain simulations, as described in the following subsections. These same simulations can be used to derive conditions for the three-dimensional co-existence of wrinkles and ridges and the steady-state propagation of ridges. In particular, the plane strain simulations enable the determination of the Maxwell condition governing the steady-state propagation of a ridge front at a critical overall compressive strain, cf., Fig. 1b. Thus, this paper will simultaneously address wrinkling/ridge transitions in the context of plane strain and three-dimensional ridge propagation and wrinkle/ridge co-existence.