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A field theory of distortion incompatibility for coupled fracture and plasticity

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ABSTRACT

The displacement discontinuity arising between the crack surfaces is assigned to smooth areal/tensorial densities of crystal defects referred to as disconnections, through the incompatibility of the continuous distortion tensor. In a dual way, the disconnections are defined as line defects terminating surfaces where the displacement encounters a discontinuity. A conservation argument for their strength (the crack opening displacement) provides a natural framework for their dynamics in the form of a transport law for the disconnection densities. Similar methodology is applied to the discontinuity of the plastic displacement arising from the presence of dislocations in the body, which results in the concurrent involvement of the dislocation density tensor in the analysis. The present model can therefore be viewed as an extension of the mechanics of dislocation fields to the case where continuity of the body is disrupted by cracks. From the continuity of the elastic distortion tensor, it is expected that the stress field remains bounded everywhere in the body, including at the crack tip. Thermodynamic arguments provide the driving forces for disconnection and dislocation motion, and guidance for the formulation of constitutive relationships insuring non-negative dissipation. The conventional Peach–Koehler force on dislocations is retrieved in the analysis, and a Peach–Koehler-type force on disconnections is defined. A threshold in the disconnection driving force vs. disconnection velocity constitutive relationship provides for a Griffith-type fracture criterion. Application of the theory to the slit-crack (Griffith–Inglis crack) in elastic and elasto-plastic solids through finite element modeling shows that it allows recovering earlier results on the stress field around cracks, and that crack propagation can be consistently described by the transport scheme. Shielding/anti-shielding of cracks by dislocations is considered to illustrate the static/dynamic interactions between dislocations and disconnections resulting from the theory. Sample size effects on crack growth are evidenced in solids encountering plastic yielding.

1. Introduction

The fundamental problem in fracture mechanics is whether a preexisting crack will grow under tractions applied at the boundary of a body. An engineering way to approach the solution to this problem is to provide a growth criterion based on

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sufficient loading intensity. In the context of linear elastic materials, the Irwin–Kies criterion postulates that a crack grows when the stress-intensity factor K, which scales the stress field at the crack tip, reaches a critical value K_c, known as the fracture toughness (Irwin and Kies, 1954). However, a more comprehensive approach involving the irreversible character of the fracture process as well as a possible extension of the crack appears as more fruitful. Indeed, appealing to the second law of thermodynamics and requiring that the dissipation in the entire body be positive is the preferred approach to establish what controls crack growth. If the material is purely elastic, the only source of dissipation is the propagating crack. Consequently, the driving force on the crack tip can be identified from the global dissipation in the entire body. It is none other than the celebrated path-independent J-integral (Eshelby, 1951; Cherepanov, 1968; Rice, 1968). The Griffith criterion then states that a crack extends if this driving force, which represents an energy release rate, becomes equal or larger than a crack growth resistance, R (Griffith, 1921). The Griffith and Irwin–Kies criteria are equivalent for linear elastic materials, because the energy release rate and stress intensity factor can be related in this case. Standard experimental methods are available to determine the crack growth resistance R and fracture toughness K_c, and it is widely accepted that these parameters are valid for arbitrary bodies, because they do not depend on the loading conditions or the sample size and geometry.

However, the large crack tip stresses invariably result in some dislocation motion in crystalline materials, as shown by *in situ* deformation in an electron microscope (Ohr, 1985), and elastic unloading is associated with plastic relaxation of internal stresses around the crack tip. Hence, crack growth necessarily implies additional dissipation in the body, in relation with dislocation motion in a more or less extended region around the crack tip. This remark suggests that crack growth occurs only if sufficient energy is available for dissipation by both crack propagation and plasticity in the bulk. Therefore, identifying the driving force for crack growth requires a global approach to dissipation, also involving the driving force for dislocation motion. In this elasto-plastic context, the J-integral is no longer path-independent, and a Griffith-type analysis encounters difficulties in assigning a driving force to crack growth (Miyamoto and Kageyama, 1978; Mura, 1987). In particular, the energy dissipation rate becomes dependent on the size and geometry of the body, which may induce a sample size dependence of the fracture toughness. Hence the transferability of the data from a test sample to an arbitrary structure may become difficult (Kolednik et al., 1997). For example, it is well known that decreasing the sheet thickness in sheet materials under plane stress results in a decrease of the fracture toughness (Bluhm, 1961; Kambour and Miller, 1981; Taira and Tanaka, 1979; Lai and Ferguson, 1986; Pardoen et al., 1999; Rivalin et al., 2001; Guo et al., 2002; Asserin-Lebert et al., 2005). Nevertheless, the engineering approach to materials and structures where crack growth is accompanied by limited plastic strain (small-scale yielding) is to treat the case as a perturbation of the linear elastic problem and to apply the Griffith and Irwin–Kies criteria, occasionally with the crack length extended by the radius of the crack tip plastic zone. Despite significant plastic dissipation in the bulk of elastic–plastic materials under large-scale yielding conditions, the common approach to crack growth in such circumstances is to use criteria based on the J-integral and the energy dissipation rate at the crack tip (Turner, 1990; Turner and Kolednik, 1994). To this end, the J-integral analysis employs the deformation theory of plasticity and regards the elastic–plastic material as non-linear elastic, which overlooks both the irreversibility of dislocation motion and the unloading phenomena close to the crack tip. In a homogeneous context, the J-integral is then found to be path-independent and it is used to assign a driving force to crack growth (Rice, 1968). More adequately in this context, the configurational forces approach (Gurtin, 1995; Maugin, 1995) allows deriving the J-integral in a dissipative setting, by using the Eshelby stress tensor instead of variational approaches or energy conservation. Following again thermodynamic guidance from dissipation arguments, this approach provides means to identify a driving force for crack growth that accounts for plasticity in the bulk (Miyamoto and Kageyama, 1978; Mura, 1987; Simha et al., 2008). As mentioned above, the J-integral is path-dependent in this context and the difference between its far field and near crack-tip values depends on the plastic activity in the bulk.

In the present paper, not unlike the configurational forces approach, plasticity is incremental and the dissipation in the entire body is envisioned in order to identify the driving forces for concurrent crack growth and dislocation motion. However, in contrast with the J-integral approach, the driving force for crack growth is not singularly supported by a point at the crack tip, but rather by a small non-zero area representing the crack tip in a regularized way. Hence, the elastic stress field is expected to be finite everywhere in the body. In addition, the internal stress field associated with the presence of dislocations in the body is accounted for, as well as its relaxation when cracks propagate. Basically, this program is grounded on the assignment of the displacement discontinuity arising between the crack surfaces to a continuous field of distortion incompatibility, through application of the Stokes theorem. In the procedure, the distortion incompatibility confers its topological meaning to a tensorial density of line defects, which we refer to as disconnections. A tautological conservation argument then provides for a natural (and kinematically undisputable) transport law for the dynamics of the disconnection densities. Following the lines of the theory of dislocation fields (Acharya, 2001), similar procedures are applied to the elastic/plastic displacement discontinuity arising from the presence of dislocations in the body, which results in involving Nye's tensor of dislocation density and dislocation transport in the analysis. The present model can therefore be viewed as an extension of the mechanics of dislocation fields to the more general context where discontinuity of the displacement field may concurrently arise from the presence of cracks.

Making connections between the present work and the so-called dislocation-based approach of fracture mechanics (Bilby et al., 1965; Barnett and Asaro, 1972; Mura, 1987; Weertman, 1996) is in order. Following hints that cracks in elastic solids could be modeled with dislocations (Eshelby, 1957; Friedel, 1964; Nabarro, 1967; Hirth and Lothe, 1982), these authors presented solutions for cracks in elastic and elasto-plastic solids that are based on dislocation density fields. It is commonly accepted however that dislocations reflect discontinuities of the elastic/plastic displacements but not the discontinuity of the *total* displacement. Therefore, the sole presence of dislocations cannot describe the discontinuity of matter inherent to fracture. In the present work, these displacement discontinuities are accounted for by the disconnections, which are crystal defects distinct from