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On force–displacement relations at contact between elastic–plastic adhesive bodies

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ABSTRACT

The loading–unloading of dissimilar adhesive elastic–plastic bodies is studied both analytically and numerically, including elastic–ideal plastic and deformation hardening behavior. The contacting bodies are assumed to be spherical in the region of contact and consequently the presented model is partly based on results pertinent to Brinell indentation. The problem of adhesive unloading is solved in two steps; first the unloading in the absence of adhesion is studied and then an adhesive pressure term is added. The analytical model is derived using fracture mechanics arguments and is based on one parameter, the fracture energy. The model is finally verified with finite element simulations by introducing a cohesive behavior between the modeled spheres. The analytical model shows very good agreement with the FE-simulations both during loading and unloading and also concerning the case of force and displacement at separation.

1. Introduction

The contact between elastic–plastic solids is a mechanical problem with a wide range of applications. Examples, among many others, are fatigue and wear of machine components which are dependent on the contact situation at the asperity level, indentation testing for extracting elastic–plastic material properties and also compaction of powders. It should be directly stated that the latter application is the one of particular interest presently.

Historically, the mechanical problem of two bodies in contact have its origin in the analysis of elastic solids by Hertz (1881) and have since then been a very active research field within the mechanics of solids. Elastic theory of contact has, after the initial contribution by Hertz (1881), made continuous progress in parallel with the developments of mathematical techniques based on for example complex variables, integral transforms and Green functions. Essential results from these analyses have been summarized by for example Gladwell (1980) and Hills et al. (1993).

Also inelastic contact problems have been analyzed frequently and in this context, especially the analyses by Johnson (1970, 1985) are worth mentioning. Important issues at inelastic material behavior include permanent as well as time-dependent deformations and stresses and are definitely of importance at analysis of the applications mentioned initially in this section.

Regardless of the importance, the complexity of inelastic contact problems makes an analytical treatment very cumbersome and most often impossible. Several nonlinearities are present such as plastic and viscous material behavior, moving contact boundaries and frictional effects and since some of these issues imply history dependence the solutions

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have to be traced incrementally. Accordingly, there are very few analytical solutions to for example plastic contact problems even though in this context the slip line analysis of rigid perfectly plastic solids by Hill (1950) should be mentioned. Accordingly, more general methods like the finite element method are most often relied upon and is nowadays a standard tool for analyzing such problems as indentation of inelastic solids. To this end, it seems natural to mention pioneering contributions by Hardy et al. (1971) and Lee et al. (1972) even though it would take an additional 15–20 years before the finite element method became a standard tool for analyzing more involved contact problems, cf. e.g. Hill et al. (1989) and Laursen and Simo (1992).

With this as a background it now seems appropriate to discuss the main application of interest presently, namely compaction of powder materials. Nowadays, compaction problems are analyzed using either a macro-mechanical method describing the powder as a continuum or a micro-mechanical approach where account is taken to deformation of the individual particles. Considering the latter approach, pioneering work was done by Wilkinsson and Ashby (1975) with further achievements by Fleck et al. (1992), Fleck (1995), Larsson et al. (1996) and Storåkers et al. (1999). These analytical models were based on some simplifying assumptions; one of them being the approximation of affine motion where the motion of each particle is determined from the macroscopic strain field. In order to relax this assumption further studies regarding quasi-static compaction of powders were made by Heylinger and McMeeking (2001), Martin et al. (2003), Martin and Bouvard (2003) and Skrinjar and Larsson (2004a,b) using the discrete element method (DEM) where each particle is modeled as a separate object. Recently, DEM has also been used to study dynamic, high-velocity compaction (Shoaib et al., 2012).

In these micro-mechanical models, the modeling of the contact behavior between the particles is critical to obtain accurate results. So far, accurate descriptions of the contact behavior exist for elastic contact following Hertz theory or for rigid plastic materials where the behavior was analyzed by Storåkers et al. (1997) and Storåkers (1997) by use of the self-similarity properties of the contact problem. In the rigid plastic self-similarity model the plastic hardening behavior is described in a power-law form and the model is only valid for two particles with the same hardening exponent or when one particle is assumed to be rigid. To relax that limitation, approximate formulae were derived and verified by Skrinjar et al. (2007) and Skrinjar and Larsson (2007) for the case when the hardening exponents of the two particles are different. These models are successful in simulating the compaction of soft materials, like bronze or aluminum, Olsson and Larsson (2012), but will not be sufficient for simulating harder materials like ceramics or cemented carbides where the elastic deformation of the contacts between the particles cannot be neglected.

In context of the discussion above, a new contact model is presently derived for elastic–plastic contacts, using results from Mesarovic and Fleck (1999, 2000) where spherical indentation of elastic–plastic solids was studied. The derived model is then verified using finite element simulations performed in the multi-purpose finite element program ABAQUS (2009).

An accurate description of the unloading and adhesive bonding of the two spheres in contact can be used for studying the elasticity and strength of pressed powder compacts. This feature been studied using DEM by Martin (2003, 2004) and later Pizette et al. (2010). The adhesion of perfectly elastic spheres is well understood; for large compliant spheres the JKR solution (Johnson et al., 1971) is applicable whereas for small stiff spheres the DMT solution (Derjaguin et al., 1975) is pertinent. The transition zone between the JKR and the DMT model was bridged by Maugis (1992) using the concept of a cohesive zone outside the contact area. Recent advances in the field of adhesion between elastic spherical bodies include measurement techniques for surface energy determination (Borodich and Galanov, 2008; Borodich et al., 2012) and the effect of surface asperities (Kesari et al., 2010; Kesari and Lew, 2011).

Studies of the adhesion between elastic–plastic spheres are comparatively rare in the literature and the studies are based on the assumption of an elastic–ideal-plastic material without any plastic hardening cf. e.g. Mesarovic and Johnson (2000) and Gu and Li (2010). The solution by Mesarovic and Johnson (2000) relies on a constant contact pressure at the onset of unloading, which is a good approximation for ideal-plastic contact, and correspondingly derived formulae for analyzing unloading in the absence of adhesion. The problem of adhesion was then solved by adding an adhesive term to the contact pressure during unloading. The same methodology will be used presently, but in this case, the contact pressure is not assumed to be constant in order to account for general strain-hardening behavior.

The accuracy of the derived model is further studied by introducing a cohesive behavior in the finite element model. Numerical studies of the adhesive contact problem is also uncommon in the literature, Radhakrishnan and Mesarovic (2009) studied adhesion between elastic spheres and Kadin et al. (2008) investigated adhesive loading and unloading of linear hardening spheres. In these studies a traction-separation law was used, derived from the Lennard-Jones potential which controls the surface atomic interactions. In this work, simplified traction-separation laws will be used based of the possibility of using a cohesive surface behavior when simulating contact in ABAQUS (2009). Unfortunately, to the authors knowledge, there are no experimental data, with all the needed material parameters specified, available for a direct comparison.

In summary then, the aim of the present analysis is to determine high accuracy predictions of the force–displacement relations at elastic–plastic contact between (locally) spherical bodies. As mentioned repeatedly above, the intended area of application concerns the analysis of the mechanics at powder compaction and in particular then when using DEM for this purpose. The whole work is driven by the need to derive easily evaluated semi-analytical expressions due to the fact that in DEM simulations the contact conditions is evaluated millions of times and thus making FE-simulations for each contact at each time step impossible even when using super-computers. The novelties of the present work are that, first of all, the