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Analysing the effects of sliding, adhesive contact on the deformation and stresses induced within a multi-layered elastic solid

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ABSTRACT

This paper presents a mathematical model of sliding, adhering contact between a rigid parabolic indenter and a multi-layered elastic solid, which is assumed to comprise of a homogeneous coating bonded through a functionally-graded transitional layer to a homogeneous substrate. The adhesive forces in this investigation are modelled using Lennard-Jones potential and an iterative algorithm is proposed that solves for the contact pressure, surface displacement and sub-surface stresses resultant within the layered solid. The effects of surface adhesion and different material properties such as varying coating/transition layer thickness and coating hardness on the solution of the contact problem are subsequently investigated in detail.

The numerical approach presented in this paper demonstrates the significance of having a suitable mathematical representation for the traction distribution along the sliding, adhering contact. It is found that under weakly adhering conditions, the assumption of only Coulombic traction suffices to determine the displacements and subsurface stresses within the multi-layered solid. However, it is noted that stress concentrations within the material begin to propagate through all three layers of the elastic solid with increased surface adhesion, which could potentially induce plasticity and lead to material ploughing under sliding. The proposed model allows us to further investigate and improve our understanding of the combined effects of traction and boundary adhesion in sliding contacts, which can be used to inform the design of materials needed in such conditions.

1. Introduction

Engineering materials that possess an increased resistance to wear can be achieved by introducing controlled gradients in the materials near their surface (Suresh et al., 1999). These graded materials, known as functionally graded materials (FGMs), exhibit either continuously or discretely varying physical and mechanical properties throughout their depth, depending on the desired application (Suresh, 2001). The graded properties of the FGM are commonly achieved through surface modification processes such as thermal spray, physical or chemical vapour deposition and laser heat treatment. The use of FGMs as protective coatings is extremely beneficial in automotive applications such as engine cylinder liners, gears and cams, as tribological studies have shown that a significant amount of surface asperity interactions during operation can induce wear, which in turn leads to a drop in mechanical efficiency (Chong et al., 2013; De la Cruz et al., 2012; Chong et al., 2014). It is therefore crucial to be able to accurately characterise and tailor graded elastic coatings to achieve the desired tribological effects that can improve wear resistance of machine elements.

A fundamental understanding of the characteristics of FGM coatings is essential when selecting materials to suit the operating conditions of the designed machine elements. This is often achieved by first using mathematical models to investigate the effects of normal loading on the contact. In the case of a homogeneous material, the classical theory of Hertz (Hertz, 1881) may be used to examine contact behaviour. However, with the miniaturisation of machine elements, short-ranged intermolecular forces will increase in significance, leading to boundary adhesion between opposing surfaces. Under these circumstances, researchers may instead choose to apply the JKR (Johnson et al., 1971), DMT (Derjaguin et al., 1975) or Maugis-Dugdale (Maugis, 1996) adhesive models to study the contact under the influence of boundary

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adhesion. Unfortunately FGMs are not homogeneous, which severely limits the usefulness of such classical models to investigate contact behaviour.

The use of FGMs as protective coatings means that a coated base material may be considered to be a layered solid. As an initial approximation, Teodorescu and Rahnejat (Teodorescu and Rahnejat, 2007) modelled a coated system as two perfectly bonded homogeneous layers and presented an iterative algorithm that may be used to compute the contact footprint within the material under normal loading. An alternative method was proposed by Chen et al. (2010) who used the Equivalent Inclusion Method (EIM) to model elasto-plastic indentation of layered materials. In a more recent development, (Yu et al., 2014) and (Wang et al., 2015) studied the frequency response functions and fretting behaviour for multilayered materials respectively, focusing on the subsurface stress propagation across the layered solid.

The studies above assumed layered solid where each layer of coating consists of homogeneous materials. For FGM coatings, the varying material properties within the graded coating may be approximated using simple mathematical functions whilst the substrate can still be considered to be homogeneous. By applying a power law to describe the graded properties within FGM layers, an analytical solution was proposed by Giannakopoulos and Suresh (1997) for a non-adhering three-dimensional graded elastic medium. Guler and Erdogan (2004) and Chidlow et al. (2011) consider the mechanical properties of the graded coating to vary exponentially throughout its depth and construct solutions to the contact problem under study using Fourier transform and Fourier series approximations respectively.

In addition to contact models developed for layered solids under normal loading, there have been models developed that seek to solve adhesive contact problems involving inhomogeneous materials. For an adhering elastic layered solid under normal loading, (Johnson and Sridhar, 2001) presented an extended JKR adhesive model. In a similar investigation, (Mary et al., 2006) proposed a semi-analytical model that describes the behaviour of a graded elastic layered solid within adhesive contacts. Sergici et al. (2006) applied a Maugis type adhesive model to investigate the contact between a spherical indenter and an elastic layered solid. More recently, (Chidlow et al., 2013) and (Chong and Chidlow, 2015) introduced surface adhesion via the Dugdale and Lennard-Jones potential respectively to simulate a rigid cylindrical geometry indenting a graded elastic layered solid comprising a FGM perfectly bonded to a homogeneous substrate.

Whilst all of the models discussed above are very useful in different circumstances, they all assume that the contact involves only normal loading, which is not sufficient to understand the tribological behaviour of a moving machine element, such as the sliding of a piston ring on an engine cylinder liner or even a cam on a tappet in an engine valve train analysis. Contact models that seek to approximate the solution of such problems must take account of combined normal loading and sliding of the contact in order to provide meaningful predictions of the behaviour of the layered elastic medium. In general, contact theories for combined normal and sliding between non-adhering surfaces are well understood with the tangential sliding given by Hamilton and Goodman's law . For example, (Hamilton and Goodman, 1966) and (Poritsky, 1950) analysed the case of a cylindrical and circular indenter sliding on the surface of a homogeneous elastic medium.

For FGMs, Giannakopoulos and Pallot looked at the contact behaviour of a flat and cylindrical punch on an elastic graded material subject to tangential and normal load (Giannakopoulos and Pallot, 2000). Guler and Erdogan (2007) and Ke and Wang (2007) also proposed mathematical models for a selection of sliding frictional contact problems involving rigid indenters of various shapes and a functionally graded material. In order to better understand the performance of graded elastic layered solids in such contacts, (Chidlow and Teodorescu, 2014) focused on characterising the influence of tangential traction induced by a cylindrical indenter on the subsurface stresses of a multi-layered elastic solid with an FGM transitional layer. For an adhering cylindrical indenter on a FGM, (Chen et al., 2009) investigated the effects of normal and tangential on the contact behaviour of their selected problem.

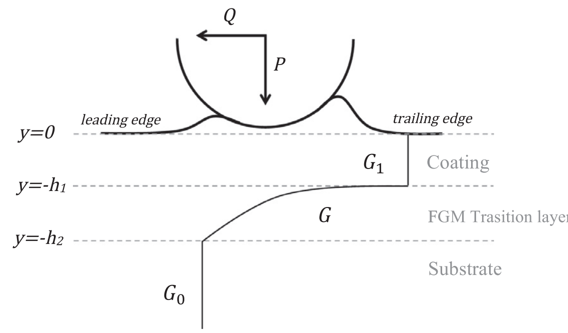

Fig. 1. Schematic diagram for the investigated contact problem.

This paper provides a thorough investigation of the effects of boundary adhesion on a rigid parabolic indenter sliding along the surface of a multi-layered elastic solid comprising a homogeneous coating bonded through a functionally-graded transitional layer to a homogeneous substrate. A previous investigation by Chidlow and Teodorescu (Chidlow and Teodorescu, 2014) showed that when the layered solid described above experiences sliding, non-adhesive contact with a rigid parabolic indenter, the magnitude of the maximum principal stresses induced within the layered solid are highly dependent on the hardness of the coating, the friction coefficient and the coating/interlayer thickness ratio. It was observed that hard coatings are particularly sensitive to the coating/interlayer thickness ratio as dramatic increases in the maximum attained stress and the location at which it occurs are observed as this parameter varies.

The work conducted within this paper uses the model derived by Chidlow and Teodorescu (2014) to describe the stresses and displacements within the layered solid induced by adhesive contact between the solid and a rigid punch using the Lennard-Jones force law. A new algorithm is then derived which allows us determine how the presence of adhesion further affects the behaviour of the stresses induced within the layered solid through contact and whether the general trends observed in the case of non-adhesive contact still hold true. To the best of the authors knowledge, the algorithm presented in this work is the only model currently available that is capable of modelling such contact problems.

2. Mathematical model

Fig. 1 illustrates the simulated contact problem, which involves a normally loaded rigid cylindrical indenter sliding over the surface of a multi-layered elastic medium in a state of plane strain. The solid occupies the lower half-plane bounded by $y = 0$ and is assumed to comprise of a finitely thick homogeneous coating occupying $-h_1 \leq y \leq 0$ (region 1) and FGM transition layer occupying $-h_2 \leq y < h_1$ (region 2) bonded to an infinitely deep homogeneous substrate (region 3). Within the transition layer, the material properties of the solid progressively change from those of the coating to those of the substrate. The shear modulus of the solid is then modelled as

$$G(y) = \begin{cases} G_1, & -h_1 \leq y \leq 0, \\ G_0 e^{\zeta(y+h_2)}, & -h_2 \leq y \leq -h_1, . \\ G_0, & -\infty < y < -h_2 \end{cases} \quad (1)$$