A constitutive hardening model of coupled plastic deformation and martensitic transformation in steels

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ABSTRACT

The article presents a constitutive model aimed at simulation of martensitic transformation induced by stress and plastic strain coupled with the hardening process in austenitic steels. The yield and transformation conditions are assumed to depend on back stresses evolving during plastic deformation and transformation processes. Their interaction affects essentially the hardening rate of a material. The thermodynamic background is reviewed and applied with both free energy and dissipation functions used in derivation of the rules of flow, hardening and transformation kinetics. The constitutive model parameters are specified and next the simulation of strain hardening for monotonic and cyclic loading is presented for the cases of uniaxial tension-compression and for combined tension-torsion deformation programs. The effect of strain ratcheting is analyzed for the cases of uniaxial and combined loading. It is shown that the transformation process is shown to essentially reduce the ratcheting strain.

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1. Introduction

The martensitic transformation occurs in various materials, such as shape memory alloys (SMA), metastable austenitic steels, low alloy multiphase steels and ceramics. The transformation is induced by varying temperature, applied stress or by plastic strain generated in monotonic or cyclic loading processes. These two irreversible processes are coupled and essentially affect the material structure and its deformational response. The growth of martensitic phase generates variation of mechanical properties, such as enhanced hardening moduli and shape variation of cyclic hysteresis loops, increased ductility and corrosion resistance, etc. During the phase transformation process generated by temperature change, the increased coupled plastic deformation (TRIP) can develop even in the nominally elastic regime, first observed by Greenwood and Johnson (1965). It was associated with the volume growth of the martensitic phase inducing plastic deformation in the weaker austenitic phase. To allow for martensite growth under an applied stress, the TRIP strain is also associated with orientation along preferred directions of martensite variants, thus generating both distortional and volumetric strains (Magee effect), see e.g. Magee (1966). The stress assisted and plastic strain assisted transformation is the first order phase transition occurring without diffusion. It generates the irreversible transformation strain result-

ing from a lattice variant of the Bain strain and the accommodating slip strain along the habit planes. The martensite growth in the thermally activated process does not exhibit the preferred orientation of habit planes. On the other hand, the stress or strain induced martensite is characterized by an oriented microstructure.

The aim of the present paper is to develop the constitutive model of a plastic deformation coupled with martensitic transformation with account for isotropic and kinematic hardening, depending on both plastic strain and martensite volume fraction. The use of internal variables allows to simulate coupled irreversible processes of plastic deformation and phase transformation within the formalism of irreversible thermodynamics. This approach allows to specify generalized thermodynamic forces resulting from the Helmholtz or Gibbs free energies. Due to plastic deformation and transformation the rate of dissipation can then be specified and expressed in terms of rates of conjugate variables. The dissipation function can then generate the dissipative forces which should be equal to free energy forces during the coupled process. The thermodynamic framework will be discussed in Section 2 and the constitutive model formulation will be presented in Section 3. It is based on works related to the concept of coupled kinematic hardening and the total back stress composed of the terms associated with both plastic deformation and phase transformation, see Mróz and Ziętek (2007) and Ziętek and Mróz (2011). Similarly, the transformation surface is introduced to specify the condition of phase growth, similar to loading-unloading conditions in plasticity.

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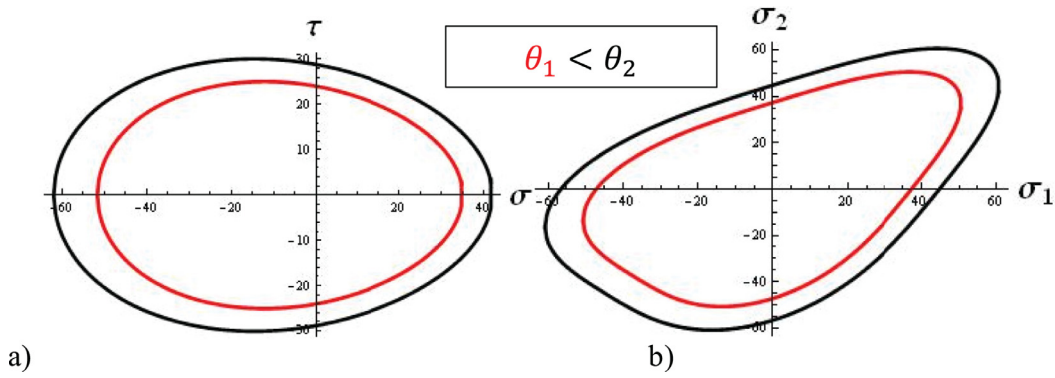 *E-mail address:* grazyna.zietek@pwr.edu.pl (G. Ziętek).

Fig. 1. Initial transformation surfaces for two temperature values: a) biaxial tension–compression, b) combined tension or compression and torsion.
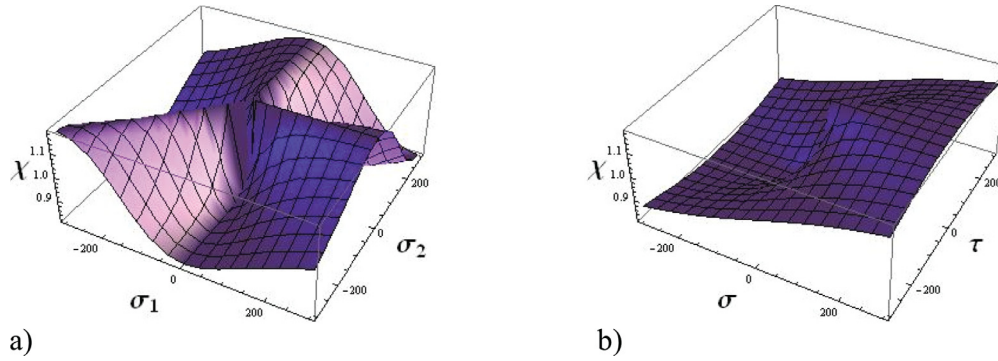

Fig. 2. Dependence of the stress factor χ on the stress components: a) biaxial tension–compression, b) combined tension or compression and torsion.

2. A review of thermodynamic framework

The authors introduced the Gibbs and Helmholtz free energies $G = G(\boldsymbol{\sigma}, \theta, \boldsymbol{\alpha})$ and $H = H(\boldsymbol{\varepsilon}^e, \theta, \boldsymbol{\alpha})$, where $\boldsymbol{\sigma}$ and $\boldsymbol{\varepsilon}^e$ are the stress and the elastic strain states, θ denotes the temperature and $\boldsymbol{\alpha}$ denotes collectively material state variables. All irreversible processes satisfy the first and second laws of thermodynamics.

$$\rho \dot{U} = \boldsymbol{\sigma} \cdot \dot{\boldsymbol{\varepsilon}} - \text{div}(\boldsymbol{q}) + r, \quad \rho \dot{S} + \text{div}\left(\frac{\boldsymbol{q}}{\theta}\right) - \frac{r}{\theta} \geq 0 \quad (1)$$

where $U = U(\boldsymbol{\varepsilon}, S, \boldsymbol{\alpha})$ is the internal energy, S denotes the entropy, $\boldsymbol{q}$ is the heat flux, r denotes the heat source and ρ is the material density. The Clausius-Duhem inequality (1) expressed in terms of the free energies takes the form:

$$\boldsymbol{\sigma} \cdot \dot{\boldsymbol{\varepsilon}} - \rho \dot{H} - \rho S \dot{\theta} - \frac{\boldsymbol{q} \cdot \nabla \theta}{\theta} \geq 0$$

$$\text{or} \quad -\dot{\boldsymbol{\sigma}} \cdot \boldsymbol{\varepsilon} - \rho \dot{G} - \rho S \dot{\theta} - \frac{\boldsymbol{q} \cdot \nabla \theta}{\theta} \geq 0 \quad (2)$$

The dot between two tensors or vectors denotes their scalar product, $a_{ij}b_{ij} = \boldsymbol{a} \cdot \boldsymbol{b}$ and ∇ denotes the space gradient. In view of the Fourier heat conduction rule there is $\boldsymbol{q} \cdot \nabla \theta \leq 0$. Neglecting the last term of (2), the entropy inequality is:

$$\boldsymbol{\sigma} \cdot \dot{\boldsymbol{\varepsilon}} - \rho \dot{H} - \rho S \dot{\theta} \geq 0 \text{ or } -\dot{\boldsymbol{\sigma}} \cdot \boldsymbol{\varepsilon} - \rho \dot{G} - \rho S \dot{\theta} \geq 0 \quad (3)$$

Now, assuming that the total strain is a sum of elastic and irreversible strains $\boldsymbol{\varepsilon}^e$ and $\boldsymbol{\varepsilon}^{ir} = \boldsymbol{\varepsilon}^p + \boldsymbol{\varepsilon}^t$, where $\boldsymbol{\varepsilon}^p$ and $\boldsymbol{\varepsilon}^t$ are the plastic and transformation strains, the inequality (3) takes the form:

$$\boldsymbol{\sigma} \cdot \left(\dot{\boldsymbol{\varepsilon}}^e + \dot{\boldsymbol{\varepsilon}}^{ir}\right) - \rho \left(\frac{\partial H}{\partial \boldsymbol{\varepsilon}^e} \cdot \dot{\boldsymbol{\varepsilon}}^e + \frac{\partial H}{\partial \theta} \dot{\theta} + \frac{\partial H}{\partial \boldsymbol{\alpha}} \cdot \dot{\boldsymbol{\alpha}}\right) - \rho S \dot{\theta} \geq 0 \quad (4)$$

or

$$-\dot{\boldsymbol{\sigma}} \cdot \left(\boldsymbol{\varepsilon}^e + \boldsymbol{\varepsilon}^{ir}\right) - \rho \left(\frac{\partial G}{\partial \boldsymbol{\sigma}} \cdot \dot{\boldsymbol{\sigma}} + \frac{\partial G}{\partial \theta} \dot{\theta} + \frac{\partial G}{\partial \boldsymbol{\alpha}} \cdot \dot{\boldsymbol{\alpha}}\right) - \rho S \dot{\theta} \geq 0. \quad (5)$$

Assuming the independent variation of $\boldsymbol{\varepsilon}^e$ and θ or $\boldsymbol{\sigma}$ and θ during the reversible process, we obtain

$$\boldsymbol{\sigma} = \rho \frac{\partial H}{\partial \boldsymbol{\varepsilon}^e}, \quad S = -\frac{\partial H}{\partial \theta},$$

$$\boldsymbol{\varepsilon}^e = \rho \frac{\partial G}{\partial \boldsymbol{\sigma}}, \quad S = -\frac{\partial G}{\partial \theta} \quad (6)$$

and the dissipation inequalities take the form:

$$\dot{D} = \boldsymbol{\sigma} \cdot \dot{\boldsymbol{\varepsilon}}^{ir} + \boldsymbol{A}_H \cdot \dot{\boldsymbol{\alpha}} \geq 0 \quad \text{or} \quad \dot{D} = -\dot{\boldsymbol{\sigma}} \cdot \boldsymbol{\varepsilon}^{ir} + \boldsymbol{A}_G \cdot \dot{\boldsymbol{\alpha}} \geq 0 \quad (7)$$

where

$$\boldsymbol{A}_H = -\rho \frac{\partial H}{\partial \boldsymbol{\alpha}}, \quad \boldsymbol{A}_G = -\rho \frac{\partial G}{\partial \boldsymbol{\alpha}} \quad (8)$$

are the generalized thermodynamic forces associated with the state variables. Now, $\dot{D}$ denotes the rate of mechanical dissipation associated with two irreversible processes, namely plastic deformation and phase transformation. Now the generalized conjugate forces are:

$$\left\{\boldsymbol{\sigma}, \ -\boldsymbol{A}_{H(G)}, \ -\frac{1}{\theta}\boldsymbol{q}\right\}. \quad (9)$$

One of the major problems now is connected with a proper selection of state variables. In all papers the martensite volume fraction ξ is taken as one of the main parameters. However, the incubation and growth of martensite occurs during cooling below the temperature $T = M_s$ and during the loading process at $T = M_s^\sigma > M_s$ with increasing stress and plastic strain. Then the isotropic martensitic structure of volume fraction ξ_θ can be distinguished from the oriented martensite structure fraction ξ_σ. These two state variables have been introduced into the form of free energy, see e.g. Leclercq and Lexcellent (1966), Bakker and Brinson (1997), Lagoudas et al. (2006). The crystallographic martensite variants $\xi^{(1)}, \xi^{(2)}, \ldots, \xi^{(n)}$ have been introduced as state variables by Turteltaub and Suiker (2006, 2005a, 2005b) and Govindjee et al. (2002, 2001). The plastic response is usually characterized by the back stress tensor and