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Research paper

A homogenization approach for effective viscoelastic properties of porous media

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ABSTRACT

The aim of this paper is to model the effective linear non-ageing viscoelastic properties of porous media based on a micromechanical approach. A porous medium is modeled as a mixture of a viscoelastic matrix and pore inclusions. The Generalized Maxwell (GM) viscoelastic model is employed for both the solid matrix and the porous medium. The effective parameters of the viscoelastic GM rheology of the porous medium, which are functions of the porosity and the viscoelastic properties of the solid phase, are derived considering the short and long term behaviors in Laplace–Carson space (LC). They are validated against exact solutions obtained from the inverse LC transform for a simple configuration. The proposed method allows avoiding the complexity of the inverse LC transform in general condition. An application for cement with assumption of spherical pore is considered to illustrate the powerful of this method.

1. Introduction

The mechanical behavior of viscoelastic heterogeneous materials is very complex due to the coupling between the non linear behavior of the phases and the heterogeneity of the whole mixture. One of the methods for analysis is the homogenization of linear viscoelastic heterogeneous media (Hashin, 1965 and 1970) and Christensen (1969), which exploits the correspondence principle between linear elasticity and linear viscoelastic (Mandel, 1966). This principle is based on the Laplace–Carson transform, which allows to convert a linearly viscoelastic problem in the time space into a linearly elastic one in the transform space. This principle is widely used to estimate the effective properties of linear viscoelastic materials, as it allows using directly in the transformed space, the classical homogenization schemes, e.g., Mori–Tanaka (Brinson and Lin, 1998; Friebel et al., 2006), self-consistent scheme (Laws and McLaughlin, 1978; Hoang-Duc et al., 2013). Effective viscoelastic properties are then derived by inverting the Laplace–Carson transform. However, the later one is not easy to carry out. A few analytical solution in some particular cases where a limited number of Maxwell chains are used to describe the matrix behavior (Thai et al., 2014), otherwise this inversion is usually performed numerically (Lévesque et al., 2006; Lahellec and Suquet, 2007; Tran et al., 2011; Gu et al., 2012). The behavior of the matrix may be represented by some different rheological models such as the generalized Maxwell model (Nguyen et al., 2015a), the generalized Kelvin model (Le et al., 2008; Nguyen, 2014) or the Burgers model (Nguyen et al., 2011).

This paper presents a micromechanical approach to estimate viscoelastic properties of porous materials that are constituted of two phases: a solid phase with generalized Maxwell behavior and pores. One interesting feature is that the expressions for both bulk and shear moduli in the time space are derived without using the inversion of the Laplace–Carson transform. Firstly, the homogenization approach for equivalent elastic behavior in LC is presented. The Mori–Tanaka scheme is applied in the LC space to estimate the mechanical parameters. Secondly, the simplest case of generalized Maxwell model with three rheological elements is considered. The results obtained for this model is then extended for general case of n elements. Effective elastic moduli and viscosity of each element are explicitly derived. The approximated solutions are compared with exact solutions obtained for a simple configuration where inverse LC transform is possible. Finally, an application for cement is considered to illustrate the powerful of the proposed method.

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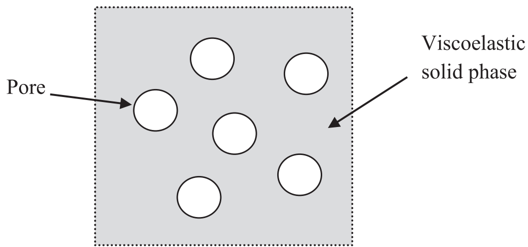

Fig. 1. Porous material containing two phases: a viscoelastic solid phase and pores.

Notations

$*$:	superscript stands for Laplace–Carson transform values
o, ∞:	under or supper scripts stand for short term and long term behaviors
t, p:	time and Laplace–Carson variables
τ:	characteristic time
$\boldsymbol{\sigma}$, $\boldsymbol{\Sigma}$:	local and macroscopic stress tensors
ε, $\mathbf{E}$:	local and macroscopic strain tensors
$\mathbb{C}$:	stiffness tensor
$\mathbb{A}$:	localization tensor
$\mathbb{J}$, $\mathbb{K}$:	spherical and deviatoric parts of fourth order unit tensor
φ:	porosity
k, μ:	bulk and shear moduli
ν:	Poisson's ratio
ηs, ηd:	bulk and shear viscosities

2. Theoretical background

2.1. Homogenization approach in LC space

A viscoelastic porous medium is modeled by a mixture of a viscoelastic solid matrix and pores (Fig. 1). The overall viscoelastic properties of such media can be obtained by considering the relationship between the local and the overall behavior of a representative elementary volume (REV) that are resumed by the following equations (Dormieux et al., 2006):

$$\boldsymbol{\sigma}^*(\underline{z}) = \mathbb{C}^*(\underline{z}) : \boldsymbol{\varepsilon}^*(\underline{z}) \tag{1}$$

$$\boldsymbol{\Sigma}^* = \mathbb{C}^* : \mathbf{E}^* \tag{2}$$

$$\boldsymbol{\Sigma}^* = \frac{1}{|\boldsymbol{\Omega}|} \int_{\boldsymbol{\Omega}} \boldsymbol{\sigma}^*(\underline{z}) d\boldsymbol{\Omega} \tag{3}$$

$$\mathbf{E}^* = \frac{1}{|\boldsymbol{\Omega}|} \int_{\boldsymbol{\Omega}} \boldsymbol{\varepsilon}^*(\underline{z}) d\boldsymbol{\Omega} \tag{4}$$

where $\boldsymbol{\sigma}^*(\underline{z})$, $\boldsymbol{\varepsilon}^*(\underline{z})$ and $\mathbb{C}^*(\underline{z})$ are the apparent local stress tensor, strain tensor and stiffness tensor, in LC space, at a point $\underline{z}$ inside the REV, respectively; $\boldsymbol{\Sigma}^*$, $\mathbf{E}^*$ and $\mathbb{C}^*$ are the apparent overall stress tensor, strain tensor and stiffness tensor of the REV, respectively; Ω is the REV and $|\Omega|$ is its volume. The local and the average strain apparent tensors are related by the following linear equation:

$$\boldsymbol{\varepsilon}^*(\underline{z}) = \mathbb{A}^*(\underline{z}) : \mathbf{E}^* \tag{5}$$

where $\mathbb{A}^*(\underline{z})$ is the apparent strain localization tensor at point $\underline{z}$. The combination of Eqs. (1)–(5) yields following equation to calculate the overall stiffness tensor of the REV:

$$\mathbb{C}^* = \frac{1}{|\boldsymbol{\Omega}|} \int_{\boldsymbol{\Omega}} \mathbb{C}^*(\underline{z}) : \mathbb{A}^*(\underline{z}) d\boldsymbol{\Omega} \tag{6}$$

In Eq. (6), the local stiffness tensor $\mathbb{C}^*(\underline{z})$ is assumed to be known, the main question is to determine the strain localization tensor $\mathbb{A}^*(\underline{z})$. Eshelby (1957) derived an analytical solution of the local tensor for the case of an ellipsoidal inclusion in an infinite homogeneous matrix. For the particular case of dry spherical pore inclusion, the apparent strain localization tensor in LC space, denoted by $\mathbb{A}^{sph*}$, is an homogeneous isotropic fourth order tensor that can be decomposed into spherical and deviatoric parts using two scalar functions of the apparent Poisson's ratio ν^* of the solid matrix:

$$\mathbb{A}^{sph^*} = \frac{3(1-\nu^*)}{2(1-2\nu^*)}\mathbb{J} + \frac{15(1-\nu^*)}{7-5\nu^*}\mathbb{K} \tag{7}$$

where $\mathbb{J} = \mathbf{1} \otimes \mathbf{1}/3$ and $\mathbb{K} = \mathbb{I} - \mathbb{J}$ are the spherical and derivatoric parts of the fourth order unit tensor $\mathbb{I}$, respectively; the tensor $\mathbf{1}$ appeared in the formula of $\mathbb{J}$ is the second order unit tensor.

The solution (7) obtained for a single pore (dilute scheme) does not take into account the interaction between the pores and then is applicable for the cases of small porosity. Such solutions can be modified using the Mori–Tanaka's scheme that considers the pore interaction by managing the boundary condition of the Eshelby's problem (Mori and Tanaka, 1973; Dormieux et al., 2006). The solutions of the Mori–Tanaka's scheme for a porous medium containing spherical pores are:

$$\frac{k^{\hom *}}{k^*} = \frac{1-\varphi}{1+Q^*\varphi} \quad \text{with} \quad Q^* = \frac{1+\nu^*}{2(1-2\nu^*)} \tag{8}$$

$$\frac{\mu^{\hom *}}{\mu^*} = \frac{1-\varphi}{1+M^*\varphi} \quad \text{with} \quad M^* = \frac{8-10\nu^*}{7-5\nu^*} \tag{9}$$

where φ is the porosity. The apparent Poisson's ratio can be calculated using the following classical relationships (as these relationships are generic, the exponent "*hom*" of the bulk and shear moduli is ignored to keep the formula simple):

$$\nu^* = \frac{3k^* - 2\mu^*}{6k^* + 2\mu^*} \tag{10}$$

It is worth noting that the choice of a homogenization scheme to model a composite material depends on its microstructure. For example Mori–Tanaka scheme is appropriate for composite materials containing a major phase that is a connected matrix of the mixture while self-consistent scheme is usually employed to model perfectly disordered materials (e.g., polycrystals). In the present work, we chose the Mori–Tanaka scheme as in Section 4 we will apply the developed method for the case of cement that can be considered as a porous material made of a percolated solid matrix and pore inclusions.

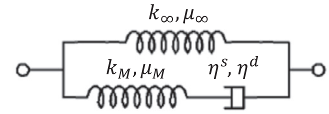

Fig. 2. Rheological standard linear solid model (Zener's model).

2.2. Standard linear solid model

Consider a rheological model of three elements (two springs and one dash-pot) as shown on Fig. 2. It is a Maxwell series in parallel with the second spring that defines the long term elastic behavior of the material. In LC space, the elastic stiffness of the equivalent elastic material is a function of the LC variable p and the viscoelastic properties of the initial material (see also Nguyen et al., 2011). For this case of three elements rheology, this dependency is expressed as:

$$\mathbb{C}^*(p) = \left(\mathbb{C}_{\boldsymbol{M}}^{-1} + \frac{1}{p}\mathbb{C}_\nu^{-1}\right)^{-1} + \mathbb{C}_\infty \tag{11}$$

where $\mathbb{C}_M$ is the elastic stiffness tensor of the spring of the Maxwell series, $\mathbb{C}_\nu$ the viscosity tensor of the dash-pot of the