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Research papaer

Dynamic crushing of cellular materials: A unique dynamic stress–strain state curve

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ABSTRACT

Cellular materials under high loading rates have typical features of deformation localization and stress enhancement, which have been well characterized by one-dimensional shock wave models. However, under moderate loading rates, the local stress–strain curves and dynamic response of cellular materials are still unclear. In this paper, the dynamic stress–strain response of cellular materials is investigated by using the wave propagation technique, of which the main advantage is that no pre-assumed constitutive relationship is required. Based on virtual Taylor tests, a series of local dynamic stress–strain history curves under different loading rates are obtained by Lagrangian analysis method. The plastic stage of local stress-strain history curve under a moderate loading rate presents a crooked evolution process, which demonstrates the dynamic behavior of cellular materials under moderate loading rates cannot be characterized by a shock model. A unique dynamic stress–strain state curve of the cellular material is summarized by extracting the critical stress–strain points just before the unloading stage on the local dynamic stress–strain history curves. The result shows that the dynamic stress–strain states of cellular materials are independent of the initial loading velocity but deformation-mode dependent. The dynamic stress–strain states present an obvious nonlinear plastic hardening effect and they are quite different from those under quasi-static compression. Finally, the loading-rate and strain-rate effects of cellular materials are investigated. It is concluded that the initial crushing stress is mainly controlled by the strain-rate effect, but the dynamic densification behavior is velocity-dependent.

1. Introduction

Cellular materials have been extensively used as core materials of anti-blast sacrificial claddings (Hassen et al., 2002; Liao et al., 2013b) and impact energy absorbers for their lightweight and superior energy absorption capability. Studying the dynamic mechanical behavior of cellular materials has become an important research direction in the field of impact dynamics. However, two coupled dynamic effects, namely inertia effect and strain-rate effect, should be taken into consideration when the dynamic mechanical behavior of materials is involved (Wang, 2005). The split Hopkinson pressure bar (SHPB) technique (Kolsky, 1949) has been developed to uncouple these two dynamic effects and the dynamic behaviors of many solid materials have been determined by this technique. Nevertheless, due to the localized deformation nature of cellular material (Deshpande and Fleck, 2000), the assumption of uniform deformation along the specimen is no longer satisfied for cellular materials under impact loading. Therefore, the application of SHPB for cellular materials under dynamic loading is still a contentious issue.

The inertia effect, which leads to stress enhancement and deformation localization as observed by Reid and Peng (1997), dominates the dynamic behavior of cellular materials under high velocity loading. According to the particular dynamic deformation features, some shock models were proposed to characterize the dynamic behavior of cellular materials. Based on a rate-independent, rigid–perfectly plastic–locking (R-PP-L) idealization, a shock model was first proposed to model the impact response of wood (Reid and Peng, 1997) and further applied to characterize the dynamic crushing behavior of metallic foams under impact/blast loading (Hassen et al., 2002; Main and Gazonas, 2008). A first-order approximation for engineering designs of cellular materials could be estimated by the R-PP-L shock model (Harrigan et al., 1999; Tan et al., 2005). A rate-independent, rigid–linear hardening plastic–locking (R-LHP-L) idealization was employed by Zheng et al. (2012) to investigate the dynamic behavior of cellular materials deformed in the shock mode and in the transitional mode. A rate-dependent, rigid–linear hardening plastic–locking

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(D-R-LHP-L) idealization was developed by Wang et al. (2013b) to study the energy conservation and critical velocities of cellular material. In order to avoid the oversimplified approximation of "locking stage" used in the above models, a rigid–power-law hardening idealization (Pattofatto et al., 2007; Zheng et al., 2013) and an elastic–perfectly plastic–hardening idealization (Harrigan et al., 2010) were further proposed. However, most of above works did not consider the loading-rate sensitivity of cellular materials. Recently, Zheng et al. (2014) proposed a rate-independent, rigid–plastic hardening (R-PH) idealization and a dynamic one (D-R-PH) to characterize the quasi-static stress–strain curve and the dynamic stress–strain states of cellular materials, respectively. Barnes et al. (2014) and Gaitanaros and Kyriakides (2014) carried out dynamic experiments and simulations of open-cell aluminum foams and investigated the Hugoniot relation of shock wave speed and particle velocity. The nonlinear plastic hardening behavior and the loading-rate effect of cellular materials under high velocity impact are much clear, but there are some different opinions in the literature (Zheng et al., 2014; Barnes et al., 2014; Gaitanaros and Kyriakides, 2014). For example, Zheng et al. (2014) reported the quasi-static and dynamic initial crushing stresses of cellular materials are different due to different deformation mechanisms, but Barnes et al. (2014) regarded that the stress ahead of the shock front is at the same level as the first local stress maximum of the quasi-static stress-strain curve. These investigations are based on the assumption of the shock-like deformation patterns, which may be improper for some impact cases, and the shock models are only suitable for the cases under high velocity loading. Thus, the dynamic behaviors of cellular materials have not been comprehensively understood, especially for the case under moderate loading rates.

Wave propagation techniques, which contain no constitutive assumption, can be used to study the dynamic behavior of materials (Wang et al., 2013a). The application potential is that the dynamic constitutive relation can be deduced directly from a series of physical quantity measurements regardless of the two coupled dynamic effects, because the interaction of the inertia effect and strain-rate effect is naturally and implicitly considered in the wave propagation technique. As a wave propagation technique, Lagrangian analysis method (Fowles and Williams, 1970; Cowperthwaite and Williams, 1971; Grady, 1973) gets the favor of researchers. However, the traditional Lagrangian analysis should consider a boundary condition, because it involves integral operations. In other words, a combination of boundary stress and particle velocity or a combination of boundary strain and particle velocity should be measured simultaneously, which requires two gauges at one position. A method combining the Lagrangian analysis and the Hopkinson pressure bar technique was proposed by Wang et al. (2011) to overcome this difficulty, and the physical quantities (stress, particle velocity, etc.) at the interface between the specimen and the pressure bar can be obtained simultaneously. Based on this technique, the "1sv+nv" and "1sε+nε" inverse analysis methods were developed according to the measured particle velocity field or strain field (Wang et al., 2011). However, these methods are not suitable for soft materials, because the boundary data cannot well match with the measured velocity data in a specimen under impact experiments. Wang et al. (2013a) proposed a much convenient method of Lagrangian analysis using the pre-known zero initial condition, but only investigated the dynamic constitutive behavior of aluminum foam under moderate velocity impact. When this Lagrangian analysis method (called "nv+T$_0$") with the Taylor-Hopkinson bar experimental device is applied, a very high impact velocity, say $v > 200$ m/s, may hardly be realized. Some other limitations, such as the accuracy of digital image correlation, also restrict the applicability of the "nv+T$_0$" Lagrangian analysis in experiment for cellular materials.

Fortunately, the finite element simulation based on cell-based models can make up the deficiencies in the experimental study, and it can offer sufficient data for theoretical analysis. Cellular materials can be well simulated by the 3D Voronoi technique (Zheng et al., 2014). By applying virtual tests, detailed and accurate data of boundary stress, nodal displacement and velocity can be obtained easily, which may hardly be measured in real experiments.

In this paper, the dynamic behaviors of cellular materials are investigated by using the Lagrangian analysis method. A brief introduction of Lagrangian analysis method is presented in Section 2. The local stress–strain response of cellular materials is determined by the Lagrangian analysis method based on the virtual Taylor test in Sections 3 and 4. The discussion on stress–strain states of cellular materials obtained by the Lagrangian analysis method is carried out in Section 5, followed by conclusions in Section 6.

2. Lagrangian analysis method

In the case of one-dimensional wave propagation, when ignoring the influences of heat conduction, body force and internal power source, mass and momentum conservation equations in Lagrangian coordinates are given by

$$\left.\frac{\partial v}{\partial X}\right|_t = -\left.\frac{\partial \varepsilon}{\partial t}\right|_X \tag{1}$$

and

$$\rho_0 \left.\frac{\partial v}{\partial t}\right|_X = -\left.\frac{\partial \sigma}{\partial X}\right|_t, \tag{2}$$

respectively, where σ, ε, v are stress, strain and particle velocity, respectively; X and t are Lagrangian coordinate and time, respectively; ρ_0 is the initial density of specimen. Here, the stress and strain are positive for compressive case, and negative for tensile case.

The mass conservation equation (Eq. (1)) establishes a relation between strain ε and particle velocity v, while the momentum conservation equation (Eq. (2)) provides a relation between stress σ and particle velocity v. Therefore, the relationship of strain and stress can be built with the aid of velocity field. Nevertheless, those quantities are connected by their first order derivatives, which means initial or boundary conditions should be provided to solve this problem.

Consider the case that the particle velocity profiles $v(X_i, t)$ at position X_i $(i=1, 2, \ldots)$ have previously been measured from numerical or experimental tests. The first order partial derivatives $\partial v/\partial X$ at time t_j $(j=1, 2, \ldots)$ and $\partial v/\partial t$ at position X_i can be numerically calculated. Hence, $\partial \varepsilon/\partial t$ at position X_i and $\partial \sigma/\partial X$ at time t_j can be indirectly obtained from Eqs. (1) and (2), respectively. Since the initial strain at $t=0$ is usually known, the strain field $\varepsilon(X_i, t)$ at position X_i $(i=1, 2, \ldots)$ can then be determined by numerical integral operation, and the stress field $\sigma(X, t)$ can be determined in the same way if the boundary stress is measured simultaneously.

However, based on the experimental study, the Lagrangian analysis methods should be combined with the path-line method, which was first introduced by Grady (1973) in order to aid the derivative computation of Lagrangian analysis. In other words, due to the incompleteness of experimental technique, the distance of two adjacent Lagrangian positions is not small enough to obtain accurate partial derivatives ($\partial \sigma/\partial X$ and $\partial v/\partial X$), and the path-line method switches the first order derivatives containing variable X to the partial derivatives containing variable t by the total differentiation along the path-line. Using the path-line method, researchers just need to know velocity profiles no less than 3 positions, and the relationship of stress and strain can be calculated by the Lagrangian analysis. The stress wave propagation characteristics in a