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Computing mobility condition using Groebner basis

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ABSTRACT

A mechanism is a set of rigid links interconnected together with ideal joints and featuring at least one degree of freedom. An overconstrained mechanism is mobile provided the links' dimensions fulfill a certain relation named the “mobility condition”. The dimensions are not independent from each other and the goal is to obtain this mobility condition. Firstly, parameters are divided into two categories: dimensional parameters and positional parameters. Dimensional parameters represent links' sizes and positional parameters represent the relative positions between the links. The closure equation models the geometric problem by capturing the relationships between the two types of parameters. The principle of the paper is to generate the mobility condition by applying Groebner basis computation to the closure equation. Three methods are presented and investigated. The first one is called the univariate polynomial method (UNIPOL); the second method is called multi-order derivation method (MOD) and the third one is called the finitely separated configurations (FISECO) method. Practical implementation of these various techniques is explained by using a standard computer algebra system. The three methods are applied on a 2D overconstrained mechanism.

1. Introduction

Degree of freedom (DOF) or mobility is a fundamental property of solids assemblies. This property is defined by the International Federation for the Promotion of Mechanism and Machine Science (IFToMM) as *the number of independent coordinates needed to define the configuration of a kinematic chain or mechanism* [1]. In [2], Gogu presents a plurality of formulas or methodologies for mobility computation. The most well-known formula is the Grubler–Kutzbach criterion. But, Muller [3] and Huang [4] explain that DOF computation fails for various reasons, mainly when studying overconstrained mechanisms. However, in industrial application, overconstrained mechanical products are often designed to withstand important forces and stresses.

The goal of this paper is to propose methods to compute algebraic relations between the dimensions of links belonging to overconstrained mechanism. This set of equations (called mobility condition) can be used during the design process of new machines or during the re-design process of existing products to help users ensuring the consistency of parameters. Indeed, when engineers want to modify one parameter of a mobile mechanism, other parameters can be calculated according to the mobility condition. Therefore, the new (or modified) mechanism is still mobile. As explained in [5–7], mobility condition can also be used during the tolerancing process in order to help users to determine, for each link, the acceptable limits of design dimensions to guaranty the mobility of all manufactured mechanisms.

To achieve this goal, two categories of design parameters are used: dimensional parameters, called u, and positional parameters, called p. Dimensional parameters are sizes of links and positional parameters are relative positions between links. They are both real

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numbers. The closure equation, modeling the geometric problem, is obtained in a classic manner. It represents the dependency between parameters u and p. The research deals with overconstrained mechanisms, meaning that the symbolic closure equation forms both an overconstrained algebraic system with respect to u and an under-constrained algebraic system with respect to p. A solution exists only if the dimensional parameters fit one (or more) relationship(s): the mobility condition. When the dimensional parameters fit the mobility condition, the closure equations form a consistent overconstrained algebraic system. It is a necessary condition to find a solution to the geometric problem.

All methods presented in this paper to compute mobility condition make use of Groebner bases. This is because this theory has been fruitfully applied to multivariate polynomial problems and also because several computer algebra systems (Maple®, Mathematica®, Magma®, Singular®, etc…) provide Groebner bases libraries.

At the outset, a generic solution to compute mobility condition is presented. This technique includes four main tasks: firstly, the initial system of equations is transformed into a system of polynomial equations; secondly, a system of univariate polynomials with respect to one positional parameter p_1 is computed using formal elimination of variables; thirdly, coefficients of p_1 are extracted and added to the system of polynomial equations and fourthly, mobility condition are achieved using again variable elimination. It is the univariate polynomial method (UNIPOL in the following). Up to our knowledge, the most similar existing technique is the Raghavan and Roth method presented in [8]. It applies to six-degree-of-freedom serial manipulators by Mavroidis in [9] and to 6–6 Stewart mechanism by Dhingra and Dongming in [10,11].

The main drawback of Groebner bases is that the number of polynomials in the resulting basis is, a priori, the exponential of the number of polynomials in the initial system [12]. In addition, the mobility condition computation is known to be NP-hard [13]. To deal with these difficulties, two new incremental technics are presented. The first one, named multi-order derivation (MOD in the following), is a generalization of the method presented by Chen in [14]. It originates from the differential properties of under-constrained equations system. The second one, named finitely separated configuration (FISECO in the following), is a closed-form version of the method presented Liu in [15]. It originates from the observation that an assembly featuring a sufficient number of distinct configurations is mobile. Finally, it should be noticed that the problem of designing dimensions so that the resulting mechanism is mobile has been tackled for a long time with geometrical methods ([16–19]) as opposed to algebraic methods.

The paper is organized as follows. Section 2 provides models: formal definition of mobility, infinitesimal mobility up to an arbitrary order and mobility through a number of specified configurations. Section 3 provides definition and properties of Groebner bases, their use for solving mobility and the foundation of UNIPOL, MOD and FISECO methods. Section 4 describes the case study, which is a planar overconstrained mechanical linkage. It also details each mobility condition computation method: UNIPOL, MOD and FISECO.

2. Mobility models

2.1. Genuine mobility

By definition, a mechanism defined by its polynomial closure equation

$$F(u,p)=0 \tag{1}$$

is mobile under the following conditions. Firstly, there exists a couple of dimensional and positional parameters (u_0, p_0) such that

$$F(u_0,p_0)=0.$$

Secondly, there exists a non degenerate arc of curve in the space of positioning parameters $t\mapsto p(t)$ such that $p(0)=p_0$, $|p'(0)|=1$ and

$$F(u_0,p(t))=0$$

for all t in a neighborhood of $t=0$. It should be noticed that $|p'(0)|=1$ does not restrict the scope of the definition because it is equivalent to $p'(0)\neq 0$ through an appropriate re-parameterization. Furthermore, since F is a polynomial, $t\mapsto p(t)$ can be supposed to be analytic.

2.2. Infinitesimal mobility

The goal of this section is to set up the general definition of infinitesimal mobility, with a special focus on first order and second order. A preliminary investigation is helpful to introduce natural definitions.

2.2.1. Investigating infinitesimal mobility

Let $F(u, p) = 0$ be the closure equation of the assembly. Let (u_0, p_0) be a couple of dimensional and positional parameters such that

$$F(u_0,p_0)=0.$$