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Viscoelastic fracture of biological composites

Eran Bouchbinder[a,*], Efim A. Brener[a,b]

[a] Chemical Physics Department, Weizmann Institute of Science, Rehovot 76100, Israel
[b] Peter Grünberg Institut, Forschungszentrum Jülich, Jülich 52425, Germany

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ABSTRACT

Soft constituent materials endow biological composites, such as bone, dentin and nacre, with viscoelastic properties that may play an important role in their remarkable fracture resistance. In this paper we calculate the scaling properties of the quasi-static energy release rate and the viscoelastic contribution to the fracture energy of various biological composites, using both perturbative and non-perturbative approaches. We consider coarse-grained descriptions of three types of anisotropic structures: (i) liquid-crystal-like composites, (ii) stratified composites, (iii) staggered composites, for different crack orientations. In addition, we briefly discuss the implications of anisotropy for fracture criteria. Our analysis highlights the dominant lengthscales and scaling properties of viscoelastic fracture of biological composites. It may be useful for evaluating crack velocity toughening effects and structure-dissipation relations in these materials.

1. Introduction

Biological composites, such as bone, dentin and nacre, exhibit outstanding mechanical properties (Arzt et al., 2003; Peterlik et al., 2006; Fratzl and Weinkamer, 2007; Ritchie et al., 2009; Dunlop and Fratzl, 2010; Ritchie, 2010; Launey et al., 2010; Ji and Gao, 2010). In particular, they combine elastic stiffness and fracture resistance that is not yet achieved by synthetic composites of similar composition. Therefore, extensive recent efforts have been aimed at exploring the basic principles underlying the heterogeneous structures and deformation mechanisms of these materials, seeking guidelines for the development of novel synthetic composites (Fratzl, 2007; Munch et al., 2008; Antonietti and Fratzl, 2010; Dunlop and Fratzl, 2010; Ji and Gao, 2010). At the nano-scale, many biological composites consist of hard, plate-like, mineral crystals embedded in a soft protein matrix. The scale and spatial arrangement of the plate-like mineral crystals are believed to play a crucial role in endowing biological composites with their remarkable mechanical properties. For example, the nanometric dimensions of the mineral crystals in bone-like composites have been proposed to be fundamentally linked to the fracture resistance of these materials (Arzt et al., 2003). Furthermore, such biological nano-composites where shown to exhibit a generic nano-structure in which the hard mineral crystals are arranged in a parallel staggered pattern inside the soft protein matrix (Fratzl, 2007; Dunlop and Fratzl, 2010; Launey et al., 2010; Ji and Gao, 2010). Other nano-composites such as nacre, composed of parallel stratified arrays of hard mineral crystals, are of great interest (Dunlop and Fratzl, 2010). Our focus in this paper is on the macroscopic implications of these nano-structures and their constitutive behaviors.

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[*] Corresponding author. Tel.: +972 8 9342605; fax: +972 8 9344123.
E-mail address: eran.bouchbinder@weizmann.ac.il (E. Bouchbinder).

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Recent effort has been devoted to the experimental characterization and modeling of the dependence of the fracture resistance of biological composites on the crack length, the so-call "crack extension resistance curve" (*R*-curve) behavior (Nalla et al., 2003, 2004, 2005; Kinney and Ritchie, 2005; Koester et al., 2008; Launey and Ritchie, 2009; Launey et al., 2010). To the best of our knowledge, much less attention has been given to the increase in fracture resistance due to the finite velocity of cracks, in spite of the fact that even small velocities may generate a non-negligible contribution to the fracture resistance (Sasaki et al., 1993; Okumura, 2002a, 2003; Ji and Gao, 2004; Iyo et al., 2004). In fact, the difference between the fracture toughness of hydrated and dehydrated biological composites may provide indirect evidence in favor of this possibility (Kruzic, 2003). This finite velocity effect can be attributed to the viscous component of the mechanical response of the soft constituent materials in biological composites (Puxkandl et al., 2002; Hazenberg et al., 2006). Our goal in this paper is to explore this possible toughening mechanism in the framework of a single timescale linear viscoelastic model for various nano-structures. We also consider fracture initiation and some related anisotropic effects.

To quantify fracture resistance, consider a crack moving at a velocity v and write the total energy release rate $G_{tot}(v)$, i.e. the amount of energy needed to create a unit of crack surfaces, as

$$G_{tot}(v) = G_0 + G_{vis}(v), \tag{1}$$

where G_0 is the velocity-independent energy release rate and $G_{vis}(v)$ is the finite-v dissipation associated with viscous deformation. Note that for $v \geq 0$ the energy release rate equals the fracture energy, $\Gamma_{tot}(v)$, which is a *materials property*, and hence G_0 should be identified with the critical energy release rate G_c, i.e. the fracture energy at the initiation of crack propagation. Nevertheless, we prefer to use the notation G_0 in Eq. (1) since it allows interpreting our results even when $G_0 < G_c$, i.e. under sub-critical conditions, where G_0 is the elastic energy release rate associated with a virtual incremental extension of the crack. Since $G_{vis}(0) = 0$, we can expand $G_{vis}(v)$ to the lowest order in v as

$$G_{vis}(v) \simeq G_0 \frac{v\tau}{d_c} w(\ell/d_c) + \mathcal{O}(v^2), \tag{2}$$

where τ is a typical viscous relaxation timescale, d_c is the smallest scale cutoff for a continuum description in a given problem, ℓ is a quantity of the dimension of length and $w(\cdot)$ is a dimensionless function. Note that even though biological composites may exhibit a hierarchy of viscous relaxation times, throughout this paper we adopt the simplifying assumption that there exists only one relevant viscous relaxation time, associated with the soft constituent materials in biological composites. Generalization to several viscous relaxation times is rather straightforward.

The generic two-dimensional fracture problem we consider in this paper consists of a crack of linear size L within a large body (i.e. a body whose linear dimensions are much larger than L) and under an applied remote tensile stress σ_∞ that tends to open it. Dimensional analysis implies that we can write G_0 as

$$G_0 = \frac{\sigma_\infty^2 L}{E} g(\overline{\ell}/L), \tag{3}$$

where E is a relevant elastic modulus, $g(\cdot)$ is a dimensionless function and $\overline{\ell}$ is a lengthscale. As implied above, the dimension of G_0 is energy per unit area.

The ultimate goal of this paper is to calculate G_{vis}, either perturbatively as in Eq. (8) or non-perturbatively as in Eq. (1), and G_0 in Eq. (3) for various composite structures. Our strategy in achieving this goal, which was strongly influenced by the work of de Gennes (1990, 2000), Okumura and deGennes (2001), and Okumura (2002a,b, 2003, 2005), is to write down coarse-grained linear elastic energy functions and viscoelastic dissipation functions for biological composites of various structures, and to use available quasi-static crack solutions to estimate in a perturbative manner the small-velocity linear viscoelastic contribution to the fracture energy. We then use "matched asymptotics" considerations to show how this analysis can be extended to a wider range of crack velocities. We focus on the scaling properties of these quantities, i.e. we systematically neglect pre-factors of order unity, and consider three types of anisotropic structures: (i) liquid-crystal-like composites, (ii) stratified composites, (iii) staggered composites, for different crack orientations with respect to the internal structure.

In ordinary isotropic viscoelastic fracture (the details are provided in Section 2.2) ℓ scales with a microscopic cutoff length (usually termed the "process zone size" Broberg, 1999), E scales with the isotropic elastic modulus and $\overline{\ell}$ scales with a macroscopic (geometric) cutoff length, the crack's length L for the fracture configuration considered here, and the functions $g(\cdot)$ and $w(\cdot)$ are of order unity. This is the hallmark of isotropic fracture: large scales elastic energy is dissipated at the small scales near the tip of a crack. Our results, summarized in Tables 1–3, show that the presence of anisotropic nano-structures introduces additional lengthscales (determined, for example, by the ratio of the elastic moduli of the soft and hard constituent materials or by the aspect ratio of the plate-like mineral crystals) gives rise to different scaling behaviors as compared to isotropic fracture. In addition, we show that anisotropy may have some implications for fracture criteria.

The structure of this paper is as follows. In Section 2 we describe the general procedure we adopt and apply it to isotropic viscoelastic fracture. In Section 3 we consider fracture in liquid-crystal-like structures. In Section 4 we consider fracture in stratified (layered) composite, for two crack orientations (parallel and perpendicular to the layers), while in Section 5 we consider fracture in staggered composites. Section 6 briefly discusses anisotropy effects and their relevance