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The frequency response of dynamic friction: Model comparisons

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ABSTRACT

Any linearised theory of the initiation of friction-excited vibration via instability of the state of steady sliding requires information about the dynamic friction force in the form of a frequency response function for sliding friction. Recent measurements of this function for an interface consisting of a nylon pin against a glass disc are used to probe the underlying constitutive law. Results are compared to linearised predictions from the simplest rate–state model of friction, and a rate–temperature model. In both cases the observed variation with frequency is not compatible with the model predictions, although there are some significant points of similarity. The most striking result relates to variation of the normal load: any theory embodying the Coulomb relation $F \propto N$ would predict behaviour entirely at variance with the measurements, even though the steady friction force obtained during the same measurements does follow the Coulomb law.

1. Introduction

Characterising the friction force at a sliding interface under the full range of possible dynamic conditions is a hard problem in constitutive modelling. Only partial solutions exist at present, arising out of the needs of particular application areas. The bulk of experimental data has been collected under conditions of steady sliding, directly relevant to questions such as the selection of materials for use in vehicle brakes and clutches, with satisfactory frictional and wear properties over a wide range of operating conditions. Self-excited frictional vibration in a system like a vehicle brake raises different concerns relating to frictional modelling. Such vibration, often called "squeal", is generally undesirable, and predictive models have been sought for many years: for a review, see for example Sheng (2008).

One class of models is associated with fully-developed stick–slip behaviour, for example in earthquake dynamics (e.g. Ruina, 1983) or a bowed violin string (e.g. Smith and Woodhouse, 2000). However, if the question of interest is the small-amplitude initiation phase of such vibration then the focus is rather different: what matters is only the dynamic behaviour of sliding friction, with no stick–slip transitions even at the asperity level. This is the main subject of this paper. Any theoretical description of friction-driven vibration must include a model for the dynamic friction force, and many proposals have been made for "laws of friction". The experimental foundation of these models is often quite limited. The purpose of this paper is to examine a new source of experimental data (Wang and Woodhouse, 2011) that provides an alternative view of dynamic sliding friction, based entirely on small modulations of the sliding speed. Measurements will be compared with the predictions of some proposed models that have found favour in recent literature. The comparison will reveal an intriguing mix of agreement and divergence. Some directions for further development of these models to produce better agreement will be explored—but it should be admitted at the outset that no convincing physically based solution will be found, and the results present a challenge for future work.

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2. Definition and measurements of $\varepsilon(\omega)$

The commonest approach to the prediction of squeal is to look for a linearised threshold of instability of the state of steady sliding, by calculating complex eigenvalues of the friction-coupled system and seeking the conditions under which one or more of these eigenvalues passes into the unstable half of the complex plane. On very general grounds it can be argued that for any such theory, the required knowledge about a friction model is rather limited. The argument is given here for single-point contact, but if the frictional contact zone is extended in space the same argument can be applied to each elemental area of the contact. A certain operating point is specified, with known values of the steady sliding speed v_0, normal load N_0, and friction force $F_0=\mu_0 N_0$ in terms of a coefficient of friction. Perhaps other environmental variables such as ambient temperature and humidity also need to be specified.

If vibration starts at very small amplitude it will result in a small oscillatory perturbation to the sliding speed v, which can be assumed to be sinusoidal in form with some amplitude v'

$$v \approx v_0 - v' e^{i\omega t} \tag{1}$$

The negative sign is introduced to match the notation of the earlier paper (Wang and Woodhouse, 2011), where it was a natural choice given the configuration of the measurement rig. This speed fluctuation v' will evoke a fluctuation in the friction force F. Under any conditions to which linearised theory can be applied, this force fluctuation must take the form

$$F \approx F_0 + F' e^{i\omega t} \tag{2}$$

with an amplitude F', which for consistency with earlier work (Butlin and Woodhouse, 2009; Duffour and Woodhouse, 2004) can be written in the form

$$F' = N_0 \varepsilon v' \tag{3}$$

The quantity ε may be complex, and will in general be a function of frequency ω. It is essentially a linearised frequency response function for sliding friction.

The predicted forms of $\varepsilon(\omega)$ for the most basic models for friction are easily written down. The oldest and most familiar "laws of friction" are usually associated with the names of Amontons and Coulomb: these state that the sliding friction force is proportional to the normal load; independent of the apparent contact area between the sliding surfaces; and independent of sliding speed. All three of these statements are open to question, especially the third. If this third statement were exactly true, then the quantity $\varepsilon(\omega)$ would simply be zero: modulation of the sliding speed would produce no change whatever in the friction force.

The next stage of historical model development was to note that measurements of dry friction force under conditions of steady sliding often reveal a coefficient of friction that varies with sliding speed. The simplest expression of this observation is to postulate that the coefficient of friction is a function of the instantaneous sliding speed: this is often called the "friction curve model" or the "Stribeck model". If the dynamic coefficient of friction μ_0 is indeed a function of sliding speed v only, ε should be real and independent of frequency: from Eq. (3) it is simply the negative of the slope of the function $\mu_0(v)$ at v_0. The precise assumed form for the function $\mu_0(v)$ makes no difference to this general conclusion; it only affects the magnitude and sign of the constant real value. This model does, of course, allow that value to vary depending on the mean sliding speed, which determines the operating point on the curve and hence the tangent slope.

One very simple model that could lead to a complex value of ε would be to assume a time delay τ between the speed variation and the friction force response. Eqs. (1)–(3) then give

$$\varepsilon = \varepsilon_0 e^{-i\omega\tau} \tag{4}$$

where ε_0 is a real constant, the value of ε as frequency tends to zero. This time-delay model allows ε to be complex, but still forces its magnitude to be independent of frequency.

A previous paper (Wang and Woodhouse, 2011) described what are believed to be the first direct measurements of $\varepsilon(\omega)$. The tests used small hemispherical pins of nylon and polycarbonate sliding against discs of glass and steel. The sliding speed was modulated at low amplitude by band-limited random noise, and the dynamic force and velocity signals measured and processed to yield $\varepsilon(\omega)$. The pin and its supporting system were designed to be very stiff, so that resonances of the measuring system do not exert any significant influence within the target frequency range 0–2 kHz: full details of the design and dynamic performance of the apparatus were given in the earlier paper. The frictional contact between the pin and the disc will be regarded as being at a single point. In reality, a well run-in pin had a contact patch with a radius of the order of 1 mm: big enough to contain multiple asperity contacts, but not big enough (it is believed) for finite-size effects such as load transfer between the leading and trailing edges to play any significant role in the dynamics. The repeatability of the measurements was shown to be good, and linearity (as measured by the coherence function from a sequence of measurements combined in an averaging procedure) was excellent. The normal load N_0 was held constant during each test. The dynamic component of normal force was measured alongside the tangential force, and was never found to show any coherent response when compared to the input velocity perturbation.

Some results for the nylon/glass combination are shown in Fig. 1: the magnitude and phase of $\varepsilon(\omega)$ are plotted for four runs with a fixed sliding speed v_0 and different normal loads N_0. These measurements, in common with other material and