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Study on spatial curve meshing and its application for logarithmic spiral bevel gears

Rulong Tan, Bingkui Chen *, Changyan Peng, Xuan Li

The State Key Laboratory of Mechanical Transmission, Chongqing University, Chongqing 400030, People's Republic of China

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ABSTRACT

The goal of the paper is to develop the theory of curving meshing and apply it in high performance gear transmission. With this aim, this study investigates the gear geometry and kinematics of space curve meshing in gear design field. Here, definition of conjugate curves and the method of obtaining them are described. What's more, an extended method to get tooth surfaces from a space curve and its conjugate curve is proposed and the meshing characteristics of these tooth surfaces are argued. Then, based on arguments for conjugate curve theorem, this paper proposes the mathematical model of conical-helix bevel gears from the idea of logarithmic spiral bevel gears. To validate this model and investigate the tooth contact characteristics of conical-helix bevel gears, the numerical example of a pair of these gears with specific profiles is represented by applying the general computer program, ANSYS 14.0. Results show that the main characteristics of these gears include loads in the same direction, pure rolling contact and low bending stresses. The results also agree with the theoretical derivations for conjugate curves and conical-helix bevel gears.

1. Introduction

In the past, researches mainly used conjugate surface theory to investigate gears. Yet, in recent years, new meshing theory has been developed. Chen et al. [1,2] proposed the method of generating the tooth surface based on the basic principle of Betrand conjugate surface. In their works, the general principle of normal circular-arc gear tooth surface was derived. Chen et al. [3,4] proposed the space curve meshing theory. In their works, the principal normal vector of a space curve was used to build meshing equation and obtained another space curve. Then, from the two space curves, they developed a new gear transmission having skewed axes named Space Curve Meshing Wheel (SCMW). Because the teeth of SCMW were constructed by tiny steel wires, they could not be used into power transmission. Chen et al. [5–7] described the method using the normal vector in any direction of a space curve to build the meshing equation. From this meshing equation, they derived the so-called conjugate curve of this known space curve. From these two curves, by introducing the method of spherical enveloping, they proposed the tooth surface design method of point-contact gears with circular-arc profile. Although the curve meshing theory has shown its advantage in gear geometry, there are some theoretical weaknesses that hinder its application. First, from present research about curve meshing, tooth profiles would be constructed by circular-tube surface. This fact limits their capacity of carrying loads. Second, in previous works, there is an absence of the arguments for meshing correctly of a pair of these tooth surfaces in theory.

Spiral-bevel gears always receive many attentions because their curved teeth contribute to smoother and quieter operation than straight-bevel gear teeth. By now, researchers have developed several types of spiral bevel gears such as circuit cut spiral bevel gears (based on Gleason's manufacturing method) [8,9] and involute spiral bevel gears (based on Klingeinberg and Oerlikon's

* Corresponding author. Tel.: +86 023 65106247.
E-mail address: bkchen@cqu.edu.cn (B. Chen).

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Nomenclature

S_i	movable coordinate systems rigidly connected to pinion ($i = 1$) and wheel ($i = 2$)
S_j	fixed coordinate systems rigidly connected to absolute space at the revolve axis of pinion ($j = 0$) and wheel ($j = p$)
Γ^1	known curve
Γ^2	conjugate curve of the known curve
S_{1Fr}	natural coordinate systems along Γ^1
Σ^i	tooth surface of pinion ($i = 1$) and wheel ($i = 2$)
ξ	shaft angle
φ	rotation angle of pinion
ϕ	rotation angle of wheel
δ	cone angle
i_{21}	transmission ratio
$\mathbf{E}_i$	shortest distance vector from the revolve axis of wheel to that of pinion in coordinate systems S_i ($i = 0,1,2$ or p)
E	magnitude of shortest distance
$\boldsymbol{v}_i^{12}$, $\boldsymbol{v}_i^{21}$	relative velocity between pinion and wheel in coordinate system S_i ($i = 0,1,2$ or p)
$\boldsymbol{r}_i^j$	position vector of known curve ($j = 1$), its conjugate curve ($j = 2$) and cross-section curve ($j = sec1, sec2$) in coordinate system S_i ($i = 0,1,2$ or p)
$\boldsymbol{S}_i^j$	position vector of tooth surface of pinion ($j = 1$) and wheel ($j = 2$) in coordinate system S_i ($i = 0,1,2$ or p)
ω_{ij}	projection magnitude of rotation velocity of pinion ($i = 1$) and gear ($i = 2$) on j-axis ($j = x,y,z$) of coordinate system S_i ($i = 1,2$)
$\mathbf{W}_i$	rotation velocity matrix of pinion ($i = 1$) and wheel ($i = 2$).
$\boldsymbol{n}_i^1$	determined normal vector of known curve in coordinate system S_i ($i = 0,1,2$ or p)
$\mathbf{n}$	common normal vector of Σ^1 and Σ^2 at contact position
$\mathbf{M}_{ij}$	coordinate transformation matrix from S_j to S_i ($i,j = 0,1,2, p, Fr1, Fr2$)
e	Euler's number
$\boldsymbol{\alpha}_i^j$, $\boldsymbol{\beta}_i^j$, $\boldsymbol{\gamma}_i^j$	the tangent vector, principal normal vector and binormal vector of known curve($j = 1$) and its conjugate curve ($j = 2$) in coordinate system S_i ($i = 0,1,2$ or p)
$\mathbf{i}^1, \mathbf{j}^1, \mathbf{k}^1, \mathbf{t}^1$	the basis vectors of x-axis, y-axis, z-axis, and homogenous coordinate axis of S_1
$\mathbf{i}^2, \mathbf{j}^2, \mathbf{k}^2, \mathbf{t}^2$	the basis vectors of x-axis, y-axis, z-axis, and homogenous coordinate axis of S_2

manufacturing method) [10], logarithmic spiral bevel gears [11]. Besides, recently, Tsai and Hsu [12] proposed a novel spiral bevel gear with circular-arc tooth profiles. Compared with other bevel gears, the logarithmic spiral bevel gear (LSBG) is an excellent transmission component because its loads are almost in the same direction. Huston and Coy [11] first investigated the geometric characteristics and model of LSBG and concluded that it's the "ideal spiral bevel gear". Then, Tsai and Chin [13] investigated the geometry of LSBG and derived the equation of logarithmic spiral in the polar form. Later on, Duan et al. [14] proposed a theory of loxodromic normal circular-arc spiral bevel gear, which can be considered as a type of LSBG because the midline of its tooth surface is a logarithmic spiral. From the idea of Wildhaber–Novikov gears, a circular arc was used to build its tooth surface. Recently, Alves et al. [15] completed the research about the design and manufacture of the logarithmic spiral bevel gears, whose tooth flanks are spherical involutes.

This work investigated the basic theory of conjugate curve meshing and developed the tooth surface generation method of space curve meshing gears. Besides, we also finished the supplementary arguments for their meshing correctly in theory. Subsequently, in the point of developed conjugate curve theory and from the idea of LSBG, the mathematical model of conical-helix bevel gear was derived and tested by a numerical sample.

2. Mathematical model of conjugate curve

In this section, the mathematical model of conjugate curve was described in detail. The extended gear design method based on the conjugate curve was proposed. Besides, the feasibility of generating tooth surfaces from a space curve and its conjugate curve was argued carefully based on the knowledge of differential geometry, matrices theory and real analysis.

2.1. The principle of space curve meshing

As for the space curve meshing theory, a pair of gear tooth surfaces can be achieved by these steps (see Fig. 1): (a) determining the first curve; (b) building the mesh equation; (c) from the known curve and mesh equation, obtaining the second curve, called conjugate curve; and (d) generating the tooth surfaces of the driving and driven gears from the two curves respectively.

There are mainly two differences between the space curve meshing and conjugate surface meshing. First, in step (b), it is required that at any point of curve, a normal vector must be determined to build the mesh equation. Second, there is an additional step, step (d). Because the curves cannot bear loads, in order to build gear solids, gear tooth surfaces need to be constructed.