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A micromechanics finite-strain constitutive model of fibrous tissue

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ABSTRACT

Biological tissues have unique mechanical properties due to the wavy fibrous collagen and elastin microstructure. In inflation, a vessel easily distends under low pressure but becomes stiffer when the fibers are straightened to take up the load. The current microstructural models of blood vessels assume affine deformation, i.e., the deformation of each fiber is assumed to be identical to the macroscopic deformation of the tissue. This uniform-field (UF) assumption leads to the macroscopic (or effective) strain energy of the tissue that is the volumetric sum of the contributions of the tissue components. Here, a micromechanics-based constitutive model of fibrous tissue is developed to remove the affine assumption and to take into consideration the heterogeneous interactions between the fibers and the ground substance. The development is based on the framework of a recently developed second-order homogenization theory, and takes into account the waviness, orientations and spatial distribution of the fibers, as well as the material nonlinearity at finite-strain deformation. In an illustrative simulation, the predictions of the macroscopic stress–strain relation and the statistical deformation of the fibers are compared to the UF model, as well as finite-element (FE) simulation. Our predictions agree well with the FE results, while the UF predictions significantly overestimate. The effects of fiber distribution and waviness on the macroscopic stress–strain relation are also investigated. The present mathematical model may serves as a foundation for native as well as for engineered tissues and biomaterials.

1. Introduction

The mechanical properties of blood vessels are fundamental for understanding hemodynamics, wave propagation, distensibility of arteries and veins, plaque stability and rupture, and vascular growth and remodeling which are strongly affected by the stress or strain of the cells. The model prediction of mechanical response of blood vessels subjected to physiological or pathological loads may help clarify the initiation, progression and clinical treatment of diseases such as atherosclerosis (Vito and Dixon, 2003). The mechanical properties of the vessels largely stem from microstructural components such as elastin and collagen fibers, cells and ground substance (Azuma and Hasegawa, 1971; Azuma and Oka, 1971; Oka, 1967, 1972; Oka and Azuma, 1970). Thus, the relation between the microstructure and macroscopic

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mechanical properties of the vessel is essential in tissue engineering, biomedical research and clinical practice. Accurate prediction of microstructural deformation and evolution, and in turn the function, will result in a new level of understanding of biological tissue in health and disease and for tissue engineering of new biomaterials.

The vast majority of constitutive models of vascular tissue are phenomenological in nature. In the pseudo-elastic models, blood vessels are modeled using constitutive relations to describe the repetitive loading and unloading behaviors separately. Hyperelasticity theory, in which the stress is calculated as the derivative of the strain energy function (SEF), has been adopted to further simplify the stress–strain relation. Fung et al. (1979) introduced a 2-D exponential SEF and later generalized it into a 3-D form (Chuong and Fung, 1983), in which the formulation applied to axisymmetric deformation of the vessel where the principal directions of the stress and strain tensors coincide with the radial, circumferential and axial directions. This leads to zero shear terms in their formulas. Deng et al. (1994) extended the formulation by adding a radial–circumferential strain to analyze axial torsion experiment and to determine the respective shear parameter. Humphrey (1995)proposed a general form of Fung-type SEF for arbitrary 3-D deformations. Other forms of hyperelastic constitutive models include the four-parameter logarithmic SEF of Takamizawa and Hayashi (1987), and the polynomial SEFs developed by Vaishnav et al. (1973) with three, seven or twelve parameters. Collectively, these models consider the vessel wall as a homogeneous material and are strictly phenomenological. Due to their simple analytic form, they have been used as a basis for investigating many vascular mechanics problems (Chuong and Fung, 1983). However, the parameters in these models have no physical meaning, and cannot be directly related to the vascular microstructure in health or disease.

There has been much effort to derive the constitutive model of the soft tissue from the geometry, distribution and the mechanical properties of the individual microstructures such as collagen and elastin fibers. There have been two major classes of micromechanical models, due to different consideration of the matrix material (cells and ground substance) in the tissue. The first class of models proposed by Lanir (1979, 1983) considers the tissue as a composite of elastin and collagen fibers embedded in a fluid-like matrix. Thus, the fibers are the only constituent phases that sustain non-hydrostatic loading such as tension and shear, while the contribution of the fluid-like matrix is a hydrostatic pressure. This assumption leads to a simplification that all the microstructures deform identically to the macroscopic deformation of the tissue since no fiber interactions are considered, such that the macroscopic SEF is the volumetric sum of fibers' SEF. On the basis of this assumption and thermodynamic consideration, Lanir (1979, 1983) developed a general multiaxial theory for the constitutive relations in fibrous connective tissues. Similarly, Decraemer et al. (1980) proposed a parallel wavy fibers model for soft biological tissues in uniaxial tension, followed by Wuyts et al. (1995). More recent developments of fluid-like matrix based models can be referred to in Refs. Dahl et al. (2008), Humphrey and Yin (1987) and Lokshin and Lanir, 2009).

A second class of micromechanical theories assume the tissue as a collagen fiber reinforced composite, whose matrix is a solid-like material that can take up loading. This assumption is motivated by the fact that the elastin, which is part of the matrix, becomes straightened and starts to take the load in the early deformation of the tissue. For example, the experimental study of Gundiah et al. (2007) suggested that the elastin can be described with a neo-Hookean constitutive model. Based on this solid-like matrix assumption, Holzapfel and Weizsacker (1998) and Holzapfel and Gasser (2000) modeled the arterial wall as a two-layer fiber-reinforced composite where the macroscopic SEF of soft tissue stems from two sources: (1) an isotropic part associated with the mechanical response of the non-collagenous matrix material (elastin fibers, cells and ground substance), and (2) an anisotropic part due to the deformation of two classes of collagen fibers symmetrically disposed with respect to the axis of the vessel. Successive developments can be found in Refs. Kroon and Holzapfel (2008), Li and Robertson (2009) and Zulliger et al., 2004. Specifically, Zulliger et al. (2004) made further refinement to account for the distribution of the waviness of collagen fibers and different SEF of the matrix and collagen fibers. These models assume *affine* deformation, i.e., the deformation of the collagen fibers and the matrix is identical to the macroscopic deformation of the tissue. Consequently, the microstructural information considered in most of these models is the volume fraction and orientation of the fibers.

There have been significant developments in the past decades to bridge the nonlinear macroscopic properties of heterogeneous non-biological media with the geometry and mechanical properties of the functional microstructures (Kailasam et al., 1997; Lebensohn et al., 2004; Liu, 2003; Liu et al., 2003; Liu and Ponte Castañeda, 2004a, 2004b; Lopez-Pamies and Ponte Castañeda, 2004a, 2004b, 2007; Ponte Castañeda, 2002; Willis, 1977). For example, the recent second-order estimate (SOE) theory of Ponte Castañeda et al. (Lopez-Pamies and Ponte Castañeda, 2004a, 2004b; Ponte Castañeda, 2002) considers the microstructural interactions, finite-strain deformation and strong material nonlinearity and heterogeneity within a framework of minimum energy principle. Applications of the model have been made to predict the macroscopic stress–strain relation and microstructural deformation of porous rubber, and show significant improvements over the previous micromechanics models when compared to finite-element (FE) simulations.

In this work, the SOE theory will be implemented to model the wavy fibrous microstructure in soft biological tissue such as vessel wall. In comparison to the affine models with a solid matrix, the present development enables consideration of full set of microstructural geometries, i.e., fiber orientation, aspect ratio, statistical spatial distribution and waviness, as well as their nonlinear interactions. The model can predict the statistically average strains in the fiber and matrix, which are heterogeneous but assumed uniform in the affine approach. This is an important feature since the accurate description of the stresses and strains in the ground substance (matrix) is important for loading-induced remodeling of the tissue. Illustrative results will be presented along with FE for validation.