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Mobility criteria of compliant mechanisms based on decomposition of compliance matrices

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ABSTRACT

Mobility analysis is an important task in the conceptual design stage of kinematic mechanisms. Not like in rigid body mechanisms, identifying the mobility of compliant mechanisms is particularly challenging as their motion is determined by both kinematic pairs and deformable compliant joints. In this paper, we investigate the use of the eigentwist and eigenwrench decomposition of compliance matrices to identify the mobility of spatial compliant mechanisms. We first prove that the eigencompliances are invariant to the coordinate transformation. We then introduce the characteristic length to scale the eigencompliances and compliance matrices to compare translational compliances with rotational ones. We propose two mobility criteria for the compliance matrix of any given compliant mechanism. We have also proposed two guidelines for choosing the characteristic length for serial open chains and closed loop mechanisms respectively. The robustness of the chosen characteristic length is discussed. A general procedure for determining the mobility for any compliant mechanism is presented. Finally, one case study is provided to verify our method.

1. Introduction

A compliant mechanism [1–3] delivers motion through elastic deformation of its flexure elements upon an appropriate load. Its benefits include no friction, no need to lubrication, compactness, and ease in manufacturability and increased precision compared with traditional rigid mechanisms. Consequently, they are used widely in various fields such as high-accuracy alignment devices for optical fibers, micro-gripper, scanning electron microscopy and precision manufacturing machine.

The mobility analysis is an important task for early design stages of mechanisms. The mobility of the traditional rigid body mechanisms can be easily identified and calculated using the well-known Glübler's formula. However Glübler's formula may fail in many cases such as singular configurations, over constrained mechanisms, redundant constraints and so on. A more general criterion would be using screw theory. In this area, numerous authors [4–6] have made significant contributions. Compared to rigid mechanisms, mobility analysis of compliant mechanisms is relatively complicated as there is no clear line to identify the degree of freedom (DOF). Howell et al. [7] presented a method to determine the DOF of the planar compliant mechanisms based on its pseudo-rigid-body-model (PRBM). Murphy et al. [8] presented a DOF formula which is the generalization of the Glübler's formula for compliant mechanisms. Based on the Murphy's formula, Ananthasuresh et al. [9] studied several cases and proposed a sequence of steps for the determination of DOF. In addition, Hopkins and Culpepper [10,11] proposed a freedom and constraint topology approach (FACT) for design of flexure mechanisms and Su [12–14] used the screw theory to analyze the mobility of

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general flexure mechanisms. However, this methodology is only qualitative in determination of degree of freedom and the dimensions of the mechanisms are assumed to be ideal. All the above proposed approaches for the mobility analysis do not take into account the effect of dimensions.

Compliance matrix itself can reveal the structure characteristic of a mechanism and many authors have investigated its geometric property and its synthesis. Lipkin and Patterson [15,16] decomposed the compliance matrix using the generalized singular eigenvalue decomposition and proposed a concept of center-of-elasticity. Patterson and Lipkin [17] described the properties of a compliance matrix in terms of several propositions based on the eigenvalue decomposition and then they [18] classified the compliance matrix based on the compliant axes. Besides, Huang et al. [19,20] studied the synthesis of a compliance matrix with simple springs connected in serial or a stiffness matrix with simple springs connected in parallel and the duality between them. Lin et al. [21] proposed a frame-invariant stiffness-based measure that is calculated from principle stiffnesses and compliances of stiffness and compliance matrices for studying quality of compliant grasps. They have also proposed the concept of equivalent stiffnesses for comparing translational and rotational stiffness of a grasp. Recently Angeles [22] made the further research about the nature of the stiffness matrix based on the screw theory and the eigenvalue problem.

In the field of compliant mechanisms, Kim [23] also apply the generalized eigenvalue decomposition to characterize basic kinematic function of a compliant mechanism and then Krishnan et al. [24] implemented the synthesis of planar compliant mechanisms at a given point based on the above paper. Yu et al. [25] also proposed an eigenscrew-based method to determine the primary mobility of compliant mechanisms and the primary mobility is the linear combination of the eigenscrew. Zou and Angeles [26] also proposed a method to study the mobility of compliant mechanisms based on the decoupling of the Cartesian stiffness matrix. This method is only applicable when the stiffness matrix can be decoupled. Both papers did not study the effects of units/dimensions on the rotational and translation DOF. Hao and Kong [27] presented a normalization-based approach to study the mobility of compliant mechanisms composed of beams. However their method is not applicable to mechanisms composed of notches and other mechanisms with a complicated topology.

In this paper, we seek to apply the generalized singular eigenvalue decomposition of a compliance matrix to determine the mobility of any compliant mechanism. The novelty of this paper lies in the use of six scaled eigencompliances to determine the number of DOF of compliant mechanisms. We also study the effects of coordinate transformation and choice of units on the mobility analysis.

The rest of the paper is organized as follows. In Section 2, we present a review on derivation of the compliance matrix of the compliant mechanism and its coordinate transformation. Section 3 reviews the eigentwist and eigenwrench characterization of the compliance matrix and study the property of invariant to coordinate transformations and how the characteristic length affects the translational eigencompliances and rotational eigencompliances. In Section 4, we present mobility criteria based on the above properties of the eigencompliances for analyzing the mobility of flexure mechanism. Section 5 gives one case study of a mechanism with varying DOF to demonstrate our method. This is followed by conclusions.

2. Background

2.1. Compliance and stiffness of flexures

A general flexure mechanism is formed by connecting a moving stage to the base through one or more flexure elements. The coordinate frame $\mathscr{F}$ is placed at the body as we are interested in the motion of the body. In this paper, we assume that the deformation is sufficiently small and within the range of elasticity.

According to the elastic theory, the relationship between the applied wrench $\hat{W}$ and the deformation twist $\hat{T}$ is

$$\hat{T} = [\mathbf{C}]\hat{W}, \quad \hat{W} = [\mathbf{K}]\hat{T}, \tag{1}$$

where $\hat{T} = \left[\vec{\theta}^T \;\; \vec{\delta}^T\right]^T$, $\hat{W} = \left[\vec{f}^T \;\; \vec{m}^T\right]^T$ are both 6 × 1 vectors. The compliance matrix $[\mathbf{C}]$ and the stiffness matrix $[\mathbf{K}]$ are 6 × 6 and $[\mathbf{C}] = [\mathbf{K}^{-1}]$ in the nonsingular case.

The work by a wrench $\hat{W}$ in the direction of the deformation twist $\hat{T}$ is calculated as their reciprocal product,

$$\hat{T}\circ\hat{W} = \hat{T}^T \; \Delta \; \hat{W} = \vec{f}^T \cdot \vec{\delta} + \vec{m}^T \cdot \vec{\theta}, \tag{2}$$

where $\Delta = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ \mathbf{I} & \mathbf{0} \end{bmatrix}$ is so-called swap operator.

Let $\hat{T}$ and $\hat{W}$ be the twist deformation and the wrench represented in the local coordinate frame $\mathscr{F}$. Their representation in a new reference coordinate frame $\mathscr{F}'$ are denoted by $\hat{T}'$ and $\hat{W}'$. They are related by

$$\hat{T}' = [\mathbf{Ad}]\hat{T}, \hat{W}' = [\mathbf{Ad}]\hat{W} \tag{3}$$

where $[\mathbf{Ad}]$ is the so-called adjoint transformation matrix given by

$$[\mathbf{Ad}] = \begin{bmatrix} \mathbf{R} & \mathbf{0} \\ \mathbf{DR} & \mathbf{R} \end{bmatrix}, \tag{4}$$