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# Input link rotatability analysis of four-bar based Watt mechanisms with revolute and prismatic joints

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ABSTRACT

This paper presents the input link rotatability analysis of four-bar based Watt six-link mechanisms, having at the most one prismatic joint. All assembly modes of such Watt mechanisms are considered in the analysis. For all the cases considered, explicit mathematical conditions are obtained, which can be used to check the input link rotatability of a given Watt mechanism. Apart from being of theoretical importance, the presented analysis is suitable for implementation within an optimization scheme, is computationally efficient and can be utilized to ensure that the synthesized mechanism does not shift from one branch to another, when put to use in practice. When incorporated within an evolutionary optimization algorithm, it can be effectively used to handle the rotatability constraint. The approach can be easily extended to Watt mechanisms containing more than one prismatic joint.


## 1. Introduction

Linkage mechanisms are widely used in practice for Function, Path and Motion Generation. The most commonly used mechanism is the four-bar mechanism. Many times, however, a four-bar may not be good enough to perform a given task. In such a situation, a six-link mechanism would be an obvious choice for the designer. All 1-DOF six-link planar mechanisms with revolute and prismatic joints (i. e., with 1-DOF joints) are inversions of the Watt and Stephenson six-link chains [1,2]. Watt and Stephenson six-link mechanisms are capable of motions exceeding the complexity of four-bar linkages, offer the benefits of simple lower-pair construction, can be designed to run at higher speeds than servomechanisms, and are durable and economical alternatives to mechanisms such as cams [3]. Many applications of Watt mechanisms, the input link rotatability of which is the topic of investigation of this paper, have been reported in literature such as: walking beam indexer with pick and place mechanism, washing machine agitator mechanism, approximate constant-velocity drag-link driven slider-crank, large-time-ratio quick-return applications [1], self-turning-off mechanism, feeding mechanism [2], double-function generation [4], 11-pose motion generation [5] and reclining chair [6]. In general, the kinematic performance of a Watt mechanism can be expected to be at par with that of a Stephenson mechanism, since they are characterized by the same number of design variables. As a result, a Watt mechanism can always be considered for many of the applications for which Stephenson mechanisms have been synthesized, such as: leader type threader [2], motion generation for a backhoe [4], function generation with single dwell [7] and complex path generation [8].

While synthesizing a mechanism, it is important to ensure that the synthesized mechanism is free from the circuit (or the branch) defect and the order defect [1,2]. Another important practical issue that commonly needs to be addressed is the rotatability of the input link of the mechanism, i.e., whether the input link is a crank or not. The input link rotatability of a four-bar can be analyzed by using the well-known Grashof criterion [1]. The input link rotatability analysis of six-link mechanisms is, however, more complex. In literature, input link rotatability of mechanisms has been studied analytically [3,9–19], more commonly from the broader perspective

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of mobility, branch, circuit and singularity analysis. Midha et al. [9] combined the concept of triangle inequality with computer-aided graphics, to formulate the mobility conditions for the planar four-bar mechanism. The approach could be extended to study the mobility of complex planar linkages. Krishnamurty and Turcic [10,11] presented a sub-Jacobian based method for the determination and elimination of branching in planar multi-loop dyadic and non-dyadic mechanisms. Chase and Mirth [12] gave precise definitions of 'circuit' and 'branch', leading to correct distinction between the two. Mirth and Chase [3] presented a quantification of circuits of all pin-jointed planar Watt six-bar mechanisms, enabling detection of the circuit defect. Ting and Dou [13] developed a method to identify the branches of Stephenson linkages, together with an algorithm to detect the branch defect. Foster and Cipra [14] addressed the problem of enumeration of circuits and branches of planar single-input dyadic (SID) mechanisms, of which the Watt mechanism is an example. Shukla and Mallik [15] specifically addressed the problem of full rotatability of the input link of six-link mechanisms and presented explicit formulae to check the existence of a crank. Ting [16] presented the concept of input joint rotation space and applied it to the five-bar, N-bar, multi-loop and spatial linkages. Ting et al. [17] presented the mobility analysis of the Watt six-bar chain. The concepts of stretch and rotation were utilized to convert a Watt six-bar chain into a degenerated Stephenson six-bar chain, enabling a unified mobility and rotatability analysis of both. Ting et al. [18] presented a unified approach based on the concept of joint rotation space, for rotatability and singularity analysis of Stephenson mechanisms and geared five-bar mechanisms. Wang et al. [19] extended the discriminant method presented in [20,21] to carry out the mobility analysis of multi-loop linkages, by algebraically implementing the concept of joint rotation space. The approaches presented in [9,16–18] can be directly used to analyze the input link rotatability of Watt and Stephenson six-link mechanisms.

Many researchers have presented synthesis of Watt and Stephenson six-link mechanisms [4–8,22–26], for various kinematic tasks. However, the analytical approaches presented in [3,9–19] were not used in any of these contributions. In [4,5], the position analysis of the candidate mechanisms was carried out for a large number of positions of the input link, to numerically check whether the input link was a crank. In [6–8,22–26], the requirement that the input link of the synthesized six-link mechanism should be a crank was not imposed. It should be noted that the analytical approaches presented in [3,9–19] provide a deeper insight into the mobility, branch, circuit, singularity and rotatability aspects of mechanisms, and can be applied or extended to more general cases of mechanisms. They are also useful in carrying out exact synthesis of defect-free mechanisms. However, these approaches are not very convenient, if one is primarily interested in analyzing a given six-link mechanism (or a number of them) with regard to input link rotatability. In such a case, an explicit formulation such as that presented in [15] is desirable.

In this paper, an explicit rotatability analysis is presented for four-bar based Watt six-link mechanisms. This work can be considered to be a significant improvement upon and extension of the work presented in [15], as follows. In [15], two different approaches were presented, depending upon whether the fixed link was a binary link or a ternary link. For one of the inversions of Watt chain wherein a binary link is fixed, no explicit formulae were obtained. Rotatability analysis was presented for Watt mechanisms containing only revolute joints. Finally, rotatability analysis for Watt mechanisms with binary fixed link and with the contained four-bar (i. e. the four-bar containing the fixed and the input links) assembled in the crossed-mode was not presented. In comparison with the analysis presented in [15], the approach presented here is uniformly applicable, irrespective of the type of the fixed link. Watt mechanisms containing one prismatic joint are analyzed in this work. Explicit formulae for all four-bar based Watt mechanisms having at the most one prismatic joint are obtained. This makes it possible to check the input link rotatability without having to carry out the position analysis even once. The approach can be easily extended to Watt mechanisms containing more than one prismatic joint. Finally, equivalence is established between Watt mechanisms with the contained four-bar operating in the crossed-mode, and Watt mechanisms with the contained four-bar operating in the open-mode.

The organization of the paper is as follows. The main contribution of this paper is presented in Sections 2–4. In Section 2, three categories of four-bar based Watt six-link mechanisms are defined, mechanisms belonging to these categories are enumerated and the input link rotatability conditions are obtained for all of them. Explicit formulae for various terms that appear in the rotatability conditions are derived in Section 3. In Section 4, the rotatability analysis for Watt mechanisms with the contained four-bar operating in the crossed-mode is presented. In Section 5, validation and advantages of the presented approach are discussed, followed by the conclusions in Section 6.

## 2. Input link rotatability conditions

### 2.1. Watt mechanisms considered for rotatability analysis

Fig. 1 shows a planar Watt six-link chain, from which a Watt six-link mechanism can be obtained by fixing any one of its links. A Watt six-link chain consists of two ternary links connected to each other, four binary links and seven 1-DOF joints. Although in Fig. 1 all the joints are shown to be revolute, some of them could as well be prismatic. From Fig. 1, it is observed that if any pair of adjacent links of the chain is chosen as the fixed link-input link pair, it would always be contained in a four-link loop. There would be two such four-link loops, if the two ternary links are chosen as the fixed and input links. For any other choices of fixed and input links, there would be only one four-link loop containing the two. From this observation and from Fig. 1, it can be said that any given Watt six-link mechanism with specified input and fixed links can be derived from a four-link mechanism containing the fixed and input links, by adding two links and three 1-DOF joints between a pair of adjacent links of the four-link mechanism. This process would result in a variety of Watt mechanisms, kinematically and also from the viewpoint of input link rotatability, depending upon the nature of joints (revolute or prismatic) within the four-link mechanism, the nature and sequence of the three joints associated with the added links and the choice of the adjacent pair of links of the four-link mechanism. From among all such possible Watt mechanisms, the mechanisms which are derived from a four-bar (i. e., a four-link mechanism with all revolute