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The kinematic synthesis of involute spiral bevel gears and their tooth contact analysis

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ABSTRACT

In this paper, a mathematical model of the spherical involute tooth surface of a spiral bevel gear whose generating curve is an arc of a circle is built. The parameters on the base cone of the gear are derived from those on the pitch cone of the gear. The algorithm derived can be used for designing the spherical involute surface of straight and spiral bevel gears. The curve of action, the paths of contact and the contact patterns of spiral bevel gears are obtained by the tooth contact analysis (TCA). The TCA results show that: 1) the curve of action is on a spherical surface; 2) the paths of contact do not change with the variation of the radii of the generating arcs of circles of the tooth surfaces of the pinion and the gear; 3) the length of the major semi axis of their contact ellipse is determined by the radii of the generating arcs of circles of the tooth surfaces of the pinion and the gear and it does not change during the transmission process. The 3-dimensional (3D) model of the gears is constructed in 3D modeling software based on the proposed method.

1. Introduction

The tooth surfaces of conventional spiral bevel gears are not exact spherical involute ones due to their manufacturing methods, in which special machine tools are used [1,2]. Although the range of the variation of the transmission ratio of the spiral bevel gears is very small, the method of the design and manufacturing of the spiral bevel gears excessively relies on the expensive machine tools. This makes the design and manufacturing of the spiral bevel gears very complicated. With the development of computer numerical control (CNC) milling technologies, the manufacturing of the spiral bevel gears may be implemented according to the exact spherical involute tooth surfaces. This will make the design and manufacturing much easier and substantially different from the conventional methods [2]. Therefore, the design and the modeling of the tooth surfaces of the spiral bevel gears are the crucial bases for the CNC milling method.

Huston and Coy approximated the tooth profile of a bevel gear on the cone surface of a back cone of the base cone of the gear instead of a spherical surface [3]. Tsai and Chin built a spherical involute profile for a straight bevel gear and gained that of a spiral bevel gear by twisting the radial straight line on the base cone of the straight bevel gear into a conic spiral curve [4]. Al-Daccak et al. derived a spherical involute equation by using spherical angles and the cone angle at any point on the tooth surface is obtained by increasing the generating angle of the spherical involute [5]. Shunmugam et al. established an exact spherical involute function by using planar angles and adopted it for building a standard straight bevel gear and a standard spiral one. They used the standard gears to analyze the normal deviation of the bevel gears cut by different machines and cutters [6]. Suh et al. presented a spherical involute equation for a straight bevel gear and then developed it for a spiral bevel gear by adding the spiral parameter [7]. Figliolini and Angeles deduced the relational expression between the base cone angle, the pitch cone angle and the pressure angle on the pitch cone for the straight bevel gears [8]. Lee et al. derived the kinematic relationships between the pressure angles, the

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pitch azimuthal angles and the pitch cone angles for the spherical involute straight bevel gears [9]. Wang et al. obtained an exact spherical involute tooth profile by using a generating-line method [10]. Park and Lee deduced an exact spherical involute function and developed a common basic rack to generate spherical involute gears for the standardization of the design and manufacturing of the straight bevel gears [11]. The meshing equation of the rack and the gear was used to explain why the curve of action of straight bevel gears is an arc of a circle. Hu et al. designed a cutting device and a cutter with an arc of a circle to cut bevel gears according to the forming principle of the spherical involute tooth surface [12]. Yang et al. proposed a method for designing the tooth profile of a spiral bevel gear based on a generating line principle and analyzed an example of the spherical involute tooth surface [13]. Zhang et al. described a generating method for a spherical involute bevel gear by using an inclined straight line on the generating surface [14].

The researches above concentrated on the design and manufacturing of the spherical involute tooth surface of straight bevel gears. However, so far there has been no method for spiral bevel gears to calculate the parameters on the base cone, such as the base cone angle and the helix angle, according to those on the pitch cone the pitch point locates, such as the position of the pitch point, the helix angle and the normal pressure angle of the tooth surface. Meanwhile, an integrated tooth contact analysis (TCA) and the contact characteristics of the spiral bevel gears need to be studied for the design of the gears.

In this paper, a mathematical model of the spherical involute tooth surface of a spiral bevel gear generated by an arc of a circle is built firstly. Then, the design formulae to calculate the parameters of the base cone are derived from those of the pitch cone and the pitch point. Furthermore, the design formulae are analyzed and verified by using two numerical examples. The curve of action, the paths of contact and the contact patterns are obtained by the TCA of the bevel gears and a 3-dimensional model of the spiral bevel gears is given. Finally, the conclusions are given.

2. Spherical involute tooth surface of a spiral bevel gear

2.1. Generation of a spherical involute and its coordinate system

In order to generate the spherical involute, the specifications of a spiral bevel gear are denoted as follows: the helix angle on the base cone is β_b; the cone angle of the base cone is δ_b; the radius of the bottom circle of the base cone is r_b; the length of the generatrix on the base cone is R_b (see Fig. 1). In the base cone, the following equation is obtained as

$$r_b = R_b \sin\delta_b. \tag{1}$$

The center of the generating circle, D, on the generating plane, T, coincides with the apex of the base cone, O_b. The bottom circle of the base cone is centered at point O_d. At the original position, the generating circle D is tangential to the generatrix on the base cone at AO_b, where point B on the generating circle D coincides with point A on the bottom circle of the base cone. When the generating circle T rolls an angle, θ, over the base cone, T is tangential to the generatrix on the base cone at NO_b and point B traces a spherical curve, AB, which is a spherical involute. The angle between BO_b and NO_b is denoted by θ_s. Because the length of the arc AN is equal to that of BN, the following equation is obtained as

$$r_b\theta = R_b\theta_s. \tag{2}$$

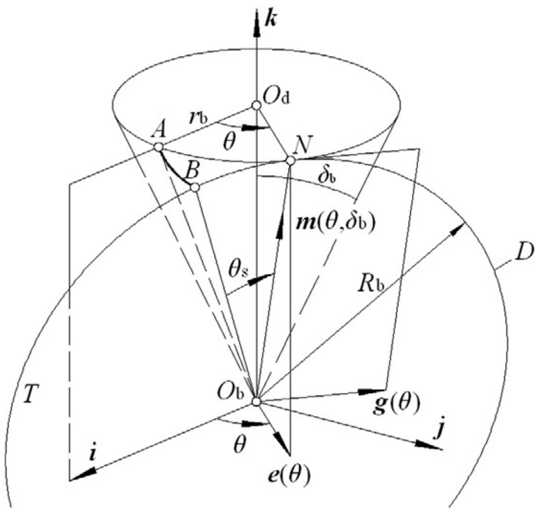

Fig. 1. Generating sketch of the spherical involute of a bevel gear and its coordinate system.