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Vibration analysis of geometrically nonlinear spinning beams

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ABSTRACT

In this paper, free vibration and primary resonances of an inextensional spinning beam with six general boundary conditions are studied. The spinning beam has large amplitude vibrations, which lead to nonlinearities in curvature and inertia. Rotary inertia and gyroscopic effects are included, but shear deformation is neglected. To analyze the free vibrations and primary resonances, the method of multiple scales is directly applied to the partial differential equations of motion. We use the concept of forward and backward mode shapes in our analyses. Expressions are derived which describe the nonlinear free vibration and primary resonances of the spinning beam with six general boundary conditions. It is shown that unlike the primary resonances in which only the forward modes are excited, in the free vibration case both forward and backward modes are excited. Numerical examples are presented for hinged–hinged and clamped–free boundary conditions and the results are verified by numerical simulations.

1. Introduction

Due to the high-speed performance of newly developed rotating machinery, the dynamical prediction and analysis of this equipment are necessary [1]. Shaw and Shaw [2] analyzed the stability and bifurcations of a balanced rotating shaft made of a viscoelastic material. They included the effects of large transverse displacements and external sources of dissipation. To examine the post-critical behaviors, the center manifold theory was applied to the nonlinear equations of motion. In addition, they examined the nonlinear forced vibration of a rotating shaft with internal damping [3]. Using the center manifold approach, they showed that the resonance was as an example of a periodically perturbed Hopf bifurcation. Choi et al. [4] presented the consistent derivation of a set of governing differential equations describing the flexural and the torsional vibrations of a rotating shaft where a constant compressive axial load was acted on it. Ishida [5] reviewed the researches on the nonlinear vibration and chaos in rotordynamics field. This paper covered the nonlinearity in restoring and damping forces such as clearances in bearings, squeeze film dampers, oil films in journal bearings, magnetic forces, seals, frictions and stiffening effects in a rotating shaft system. Kurnik [6] analyzed the stability and self-excited postcritical whirling of a rotating shaft with the aid of bifurcation theory. The shaft was made of a material with elastic and viscous nonlinearities. He derived the equations of motion by neglecting rotary inertia and effects of Von-Karman nonlinearity. But, he considered the curvature nonlinearity. The primary resonance of a non-linear rotor system was considered by Cveticanin [7]. The nonlinearity was due to elastic material properties. The rotor system was modeled by a two-degree-freedom system. To analyze the system, an averaging method was applied. The analytical solutions were validated by experiments. Sturla and Argento [8] studied the free and forced response of a viscoelastic spinning Rayleigh shaft.

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List of symbols

A	cross section area
A_{11}	longitudinal stiffness
c	external damping coefficient
D_{11},D_{22}	torsional and flexural stiffness
e	strain along the centroidal axis of the beam
E	elasticity modulus
e_ς, e_ξ	eccentricity distributions with respect to axes y and z
G	shear modulus
I_1,I_2	polar and diametrical mass moment of inertia
l	length of spinning beam
m	mass per unit length of beam
$Q_v(x, t)$, $Q_w(x, t)$	excitations in two orthogonal transverse planes
u	longitudinal displacement
v,w	transverse displacements
ε	dimensionless small ordering parameter
$\phi(x,t)$	torsional deformation
$\phi_b(x)$	backward mode shape
$\phi_f(x)$	forward mode shape
γ_b	linear backward natural frequencies
γ_f	linear forward natural frequencies
ρ	mass density
$\rho_i(i = 1 - 3)$	beam curvatures
σ	detuning parameter
Ω	spinning speed
Ψ,θ,β	3–2–1 Euler angles

They derived a closed-form polynomial frequency equation and integral expressions for the response to a general forcing function. Nonlinear forced oscillations of a rotor with distributed mass were discussed by Ishida et al. [9]. The geometric nonlinearity in the rotor was due to the stretching of the rotor center-line. It was shown that the primary resonance curve is of a hard spring type and that only some kinds of combination resonances may occur. Ishida et al. [10] investigated the nonstationary vibrations of a nonlinear rotating shaft during accelerating through a major critical speed. They used the asymptotic and complex-FFT method to analyze the system. It was shown that in the nonstationary case, the first approximation of the asymptotic method generates large errors. Ishida and Inoue [11] analyzed the influence of internal resonance on the nonstationary vibrations of a nonlinear rotor during the acceleration through the major critical speeds. In fact, the existence of an internal resonance made an easy passing through the major critical speed of a rotor system. Melanson and Zu [12] studied the free vibration and stability analysis of internally damped rotating shafts with general boundary conditions. They used Timoshenko beam theory and considered the effects of internal viscous and hysteretic dampings. The free and forced vibration analysis of a rotating disk–shaft system with linear elastic bearings was investigated by Shabaneh and Zu [13]. Bearings were mounted on viscoelastic suspensions. Shaft was modeled by Timoshenko beam theory and the viscoelastic support was modeled as a Kelvin–Voigt model. A geometrically non-linear model of a rotating shaft was introduced by Luczko [14]. The model included Von-Karman non-linearity, non-linear curvature effects, large displacements and rotations as well as gyroscopic and shear effects. To solve the system, he used the Galerkin and continuation methods and analyzed the internal resonances. The critical speeds and mode shapes of a spinning Rayleigh beam with six general boundary conditions are investigated analytically by Sheu and Yang [15]. Viana Serra Villa et al. [16] used the invariant manifold approach to explore the dynamics of a nonlinear rotor. They constructed a reduced order model with the aid of nonlinear normal modes and evaluated its performance. Cveticanin [17] considered the free vibration of a Jeffcott rotor with cubic nonlinear elastic property. He applied the Krylov–Bogolubov method to solve the nonlinear equations of motion. Dimentberg et al. [18] studied the response of a Jeffcott rotor with external and internal damping to random excitation, where mean square responses were predicted by the method of moments. Free vibrations of a rotating beam with random properties [19], and vibrations and reliability of a rotating beam with random properties under random excitations [20] were studied by Hosseini and Khadem. In addition, we analyzed the different dynamical behaviors of rotors in several articles [21–26].

A spinning beam can be used to model turbine shafts, spinning missiles, and other mechanical elements. The linear analysis of a spinning beam with general boundary conditions are studied previously [15]. Nevertheless, there is no research about the nonlinear vibrations of spinning beams with general boundary conditions. In this paper, free vibrations and primary resonances of a geometrically nonlinear spinning beam with six general boundary conditions are studied. The spinning beam has large amplitude vibrations, which cause nonlinearities in curvature and inertia. Rotary inertia and gyroscopic effects are included, but