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Short communication

Analytical determination of back-side contact gear mesh stiffness

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ABSTRACT

Back-side gear tooth contact happens when anti-backlash (or scissor) gears are used, tooth wedging or tight mesh occurs, or vibration amplitudes are high enough that teeth separate and pass the backlash zone. An accurate description of the back-side gear tooth mesh stiffness is needed to study gear mechanics in such cases. This work studies the time-varying back-side mesh stiffness and its correlation with backlash by analyzing the relationship between the drive-side and back-side mesh stiffnesses. Results of this work yield the general form of the back-side mesh stiffness in terms of the known drive-side mesh stiffness for an arbitrary gear pair. The analytical results are confirmed by simulation results from gear contact analysis software that precisely tracks drive- and back-side gear tooth contact.

1. Introduction

Back-side contact in a gear mesh refers to contact on the surfaces of a gear that are not used to transmit power. Recent studies on gear dynamics [1–4] show that it is possible for tooth wedging (or tight mesh), that is, simultaneous drive-side and back-side contact, to happen in applications such as wind turbine gearboxes. Tooth wedging in wind turbines results from the combined effect of gravity and bearing clearance nonlinearity, and it proved a likely source of gearbox bearing failures in a particular case. For better understanding of the impact of tooth wedging on gearbox failures, it is necessary to have a model that includes accurate description of the back-side contact mesh stiffness.

Besides tooth wedging, anti-backlash (or scissor) gears are another case for back-side contact to occur. To minimize the undesirable characteristics caused by backlash, anti-backlash gears eliminate the backlash by using a preloaded spring to force the fixed part of the driving gear to contact the drive-side of the driven gear teeth and, simultaneously, force the free part of the driving gear to contact the back side of the driven gear [5]. Accurate modeling of back-side contact mesh stiffness is necessary to analyze such systems.

Mesh stiffness variation and its impact on gear mechanics have been extensively investigated. Mesh stiffness variation is the source of static transmission error fluctuations. Munro and his team experimentally investigated gear tooth mesh stiffness throughout and beyond the path of contact [6]. Blankenship and Kahraman experimentally and analytically studied a single degree of freedom gear pair driven by time-varying mesh stiffness variation; they showed contact loss and back-side contact that are subject to a symmetric backlash condition [7]. The same system was investigated with analytical and finite element models [8]. Lin, Liu, and Parker analyzed mesh stiffness variation instabilities in two-stage gear systems [9,10], as well as in simple planetary gear systems [11]. Their studies showed that parametric excitation from time-varying mesh stiffness causes instability and severe vibration under certain operating conditions. They applied a perturbation method to analytically determine the instability conditions. Velex and Flamand extended the

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research scope to planetary gear trains and studied their dynamic responses with varying mesh stiffness [12]. Wu and Parker [13] extended the study on parametric instability to planetary gears with elastic continuum ring gears. Sun and Hu [14] investigated mesh stiffness parametric excitation and clearance nonlinearity for simple planetary gears. Bahk and Parker [15] derived closed-form solutions for the dynamic response of planetary gears with time-varying mesh stiffness and tooth separation nonlinearity based on a purely torsional planetary gear model. They extended this to systems with tooth profile modifications [16]. Guo and Parker [1] modeled and analyzed a simple planetary gear with time-varying mesh stiffness, tooth wedging, and bearing clearance nonlinearity. Although back-side contact is included in their model, the average value of the periodic mesh stiffness on the drive-side is used to approximate the back-side mesh stiffness, which is a simplified description of the back-side mesh stiffness.

Despite the abundance of literature on mesh stiffness variation and gear dynamics, no studies have derived the back-side mesh stiffness in their analytical model. One possible reason is that the usual symmetry of the gear teeth ensures that the contact ratios, mesh periods, and average mesh stiffnesses over the mesh period are the same for drive- and back-side contact. This may lead to the mistaken conclusion that the back-side mesh stiffness is the same as the drive-side one. For example, Kahraman and Blankenship performed experiments on the nonlinear response of spur gear pairs with varying involute contact ratios [17,18]. The back-side contact is assumed to be the same as the drive-side contact in their study. In the tight mesh case shown in Fig. 1, the back-side mesh stiffness, however, is not equivalent to the drive-side one, because the back-side contact is along the back-side line of action (the dashed line in Fig. 1) and the number of gear teeth in contact along the back-side line of action is not always equal to that along the drive-side line of action (the solid line in Fig. 1). Fig. 2 illustrates one such case (the simulation results are from Calyx [19], a multi-body finite element/contact mechanics program with precise gear tooth contact capability). There are two pairs of gear teeth in contact along the back-side line of action, while only one pair of teeth is in contact along the line of action. Therefore, the back-side mesh stiffness differs from the drive-side mesh stiffness at this moment.

2. Derivation of back-side mesh stiffness

The drive-side mesh stiffness refers to the stiffness of the nominally contacting teeth at a mesh in the direction of power transmission. It varies as the number of teeth in contact fluctuates with the gear rotation. The stiffness acts along the line of action. The period of its variation is known for the given rotation speed. Mesh stiffness variation functions are often approximated by Fourier series in analytical studies. They can be accurately calculated by finite element software. The drive-side mesh stiffness function is critical for analytical studies on gear dynamics [8,15,20–22].

Similar to the drive-side mesh stiffness, the back-side mesh stiffness is the stiffness of the changing number of contacting teeth along the back-side line of action. In order to have simultaneous drive-side and back-side contacts at all times, we first investigate an ideal gear pair (one that operates at the nominal center distance and has zero backlash tooth thickness). The results of the ideal gear pair are then extended to arbitrary gear pairs with backlash and to anti-backlash gears.

2.1. Back-side mesh stiffness for an ideal gear pair

Fig. 3 illustrates the drive-side and back-side contacts for an ideal gear pair. The gear details are not needed for what follows. The tooth numbers of the driving and driven gears are Z_{dr} and Z_{dn}, respectively. T is the mesh period. At $t = 0$, the pitch point at the drive-side of the driving gear is in mesh (Fig. 3a). The dashed line in the middle of each sub-figure is the center line between the two gears. After one mesh period T, the driving gear tooth moves one driving gear circular pitch $p_{dr} = \frac{2\pi r_{dr}}{Z_{dr}}$, and the driven gear tooth moves $p_{dn} = \frac{2\pi r_{dn}}{Z_{dn}}$ (r_{dr} and r_{dn} are the pitch radii). After one fourth of the mesh period ($t = \frac{T}{4}$), the driving gear tooth moves one fourth of its circular pitch. Because there is no backlash along the pitch circle, the circular tooth thickness of the driving gear q_{dr} is

$$q_{dr} = \frac{1}{2} p_{dr}. \tag{1}$$

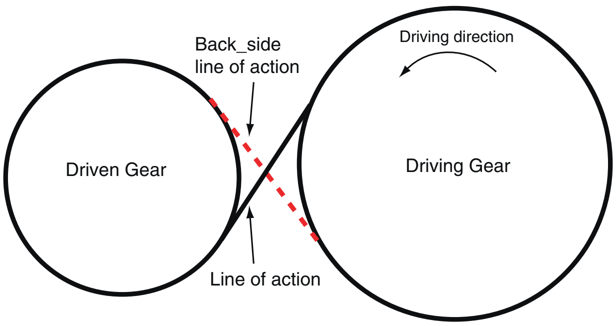

Fig. 1. Drive-side gear contact (solid line) and back-side gear contact (dashed line) in a tight mesh case (both drive and back-sides are in contact).