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3-D waves in porous piezoelectric materials

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ABSTRACT

Wave propagation in porous piezoelectric materials, possessing crystal symmetries monoclinic (2, m), orthorhombic (222, 2 mm), tetragonal (4), trigonal (32), hexagonal (6 mm) and cubic ($\bar{4}$3 m), is studied. The Christoffel equation is derived for 3D waves in an anisotropic porous piezoelectric medium. The four roots of the biquadratic equation give the complex wave velocities of four waves propagating in such a medium. These complex wave velocities are resolved to obtain the phase velocities and attenuation factors of waves. The algebraic implicit expressions are derived for monoclinic (2, m), orthorhombic (222, 2 mm), tetragonal (4), trigonal (32), hexagonal (6 mm) and cubic ($\bar{4}$3 m) crystal classes. The characteristics of waves in porous piezoelectric materials are studied in terms of the velocity surfaces and attenuation surfaces. The effects of phase direction, frequency, piezoelectricity, porosity and crystal symmetry on the velocity surfaces and attenuation surfaces are investigated for these crystal classes. The effects of phase direction and crystal symmetry on the skewing angles and wave fronts of quasi waves are also studied.

1. Introduction

Piezoelectric materials possess the important property of linear coupling between mechanical and electrical fields, which renders them useful as transducers, actuators, sensors and filters, etc. in wide range engineering applications in smart structures and devices. Wave propagation in piezoelectric media has been extensively investigated in connection with the generation and transmission of disturbances in electro-acoustic devices such as transducers and resonators. Some of the notable texts on wave propagation in piezoelectric materials are Auld (1973, 1990); Nayfeh (1995) and Royer and Dieulesaint (1996). A detailed survey of wave propagation in piezoelectric materials belonging to different crystal classes is presented in the texts (Auld, 1973, 1990; Royer and Dieulesaint, 1996). Three waves viz. quasi P, quasi S_1 and quasi S_2 waves propagate in anisotropic piezoelectric materials. One of the essential features of the wave phenomenon in such materials is the stiffened waves which exist as a result of electro-mechanical coupling. The phase velocities and slowness do depend upon directions of propagation, crystal symmetry, poling directions and other features of the medium. Kyame (1949) studied the wave propagation in piezoelectric materials by taking quasi-static electric field approximation into account. The propagation of waves at large distances from a source of disturbance in an infinite piezoelectric medium of hexagonal symmetry was investigated by Rao (1978). Auld (1981) presented a short survey related to wave propagation and resonance phenomena in piezoelectric materials and relates the concepts and theory to the physical properties and crystal symmetry of the materials. Special features peculiar to wave propagation in piezoelectric materials were noted and a brief sketch of the methods used for solving piezoelectric boundary value problems was also given. It was found that an extra

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solution appears due to piezoelectric stiffening terms when only one of the components of the wave vector is specified. Christoffel equations for electroacoustic waves in unbounded piezoelectric crystals were solved by Every (1987) for the weak and strong electromechanical coupling cases. In general, waves in piezoelectric materials are quasi waves. However, for some types of material symmetry, pure longitudinal or pure transverse wave modes exist for certain specified directions of propagations (Romeo, 1996). Daros and Antes (2000) derived the strong ellipticity conditions for piezoelectric materials of the crystal symmetry classes 6 mm and 622 using positivity conditions for quadratic, cubic and quartic polynomials. The differential equation for the wavefront shape due to a line source in cylindrically hexagonal piezoelectric solids was obtained by Daros (2000).

In spite of the great advances in the development of single phase piezoelectric materials with high hydrostatic sensitivity, such materials have shortcomings such as large hydrostatic piezoelectric coefficients and flexibility, high hydrostatic sensitivity and low acoustic impedance, etc. However, due to high hydrostatic figure of merit and low acoustic impedance, porous piezoelectric materials have been of great interest and technological importance in ultrasonic applications such as hydrophones, actuators, miniature accelerometer and underwater transducers (Shrout et al., 1979; Arai et al., 1991). Use of the piezoelectric effect in porous piezoelectric ceramics offers an original method for studying the coupling between the electrical, mechanical, permeability and of course piezoelectric properties of porous systems.

Different analytical models (Matsunaka et al., 1988; Banno, 1993; Gomez and Montero, 1996a) have been developed to study the effects of pore volume fraction and the connectivity on the elastic, dielectric and piezoelectric properties of porous piezoelectric materials. Experimental studies (Roncari et al., 2001; Xia et al., 2003; Boumchedda et al., 2007; Levassort et al., 2007; Zeng et al., 2007; Lee et al., 2008; Wang et al., 2008; Boumchedda et al., 2010) have been done related to characteristics, fabrication and manufacturing of porous piezoelectric materials and the influence of porosity on its properties. The study of the effects of porosity on the electromechanical properties of porous piezoelectric materials, on the basis of numerical models based on finite-element method, was done by Kar-Gupta and Venkatesh (2006, 2007, 2013).

Lacour et al. (1994) presented a theory to study the effects of compressibility and permeability of the viscous fluid saturating the porous piezoelectric materials on the piezoelectric properties of piezoelectric ceramics. Gomez and Montero (1996b, 1997a,b) dealt with a new system of constitutive relations that describe the elastic, dielectric and the piezoelectric behavior of porous piezoelectric materials. The influence of the coupling mechanisms on the material parameters was also analyzed. Craciun et al. (1998) made an experimental study on wave propagation in porous piezoelectric ceramics in order to improve their properties over dense ceramics. The effects of porosity on phase velocity and attenuation of longitudinal waves were also studied experimentally. Gomez et al. (2000) adopted a 2D model to study the propagation of acoustic plane waves through composite materials using the finite-element method. Altay and Dokmeci (2005) expressed the governing equations of a porous piezoelectric continuum in variational form and obtained a three field variational principle with some conditions. A survey of the literature related to porous piezoelectric materials reveals that very few authors have established theoretical models for porous piezoelectric materials. Vashishth and Gupta (2009a) derived the constitutive equations and equations of motion for porous piezoelectric materials. In order to improve the properties of porous piezoelectric materials, the knowledge of their physical properties and wave phenomena in such a medium is desirable. An analytical study of wave propagation in transversely isotropic porous piezoelectric materials was done by them (Vashishth and Gupta, 2009b). The effects of porosity, direction of propagation and piezoelectricity on the phase velocities, slowness and attenuation coefficients were investigated therein. Sharma (2010) analyzed the effects of piezoelectricity on the phase velocities and group velocities of waves propagating in an anisotropic porous piezoelectric material. Subsequently, the uniqueness theorem, theorem of reciprocity and general theorems in the linear theory of porous piezoelectricity were established (Vashishth and Gupta, 2011a). Based on the theoretical formulation developed, Vashishth and Gupta, 2011b, 2012, 2013 carried out studies of reflection and transmission of waves in models involving porous piezoelectric materials.

Three-dimensional wave propagation in porous piezoelectric materials, for different symmetry classes, is studied in this paper. The porous piezoelectric material is assumed to be saturated with a viscous fluid. The Christoffel equation, corresponding to an anisotropic porous piezoelectric medium, is derived. The wave velocities of four inhomogeneous waves are obtained. These wave velocities give further the phase velocities and attenuation coefficients of the corresponding waves. The results for different crystal classes, viz. monoclinic (2, m), orthorhombic (222, 2 mm), tetragonal (4), trigonal (32), hexagonal (6 mm) and cubic ($\bar{4}3$m) are deduced. The effects of phase direction, frequency, piezoelectricity, porosity and crystal symmetry on the velocity surfaces and attenuation surfaces are investigated. The effects of phase direction and crystal symmetry on the skewing angles and wave fronts are also studied.

2. Christoffel equation for anisotropic porous piezoelectric materials

A porous piezoelectric material saturated with a viscous fluid is considered. The constitutive equations (Vashishth and Gupta, 2009a) for anisotropic porous piezoelectric materials are

$$\boldsymbol{\sigma} = \mathbf{c}.\boldsymbol{\varepsilon} + \mathbf{m}\boldsymbol{\varepsilon}^* - \mathbf{e}.\mathbf{E} - \boldsymbol{\zeta}.\mathbf{E}^*, \tag{1}$$

$$\boldsymbol{\sigma}^* = \mathbf{m}.\boldsymbol{\varepsilon} + \mathbf{R}\boldsymbol{\varepsilon}^* - \tilde{\boldsymbol{\zeta}}.\mathbf{E} - \mathbf{e}^*.\mathbf{E}^*, \tag{2}$$