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Study of dynamics of soft contact rolling using multibond graph approach

Anil Kumar Narwal [a], Anand Vaz [b,*], K.D. Gupta [a]

[a] *Mechanical Engineering Department, Deenbandhu Chhotu Ram University of Science and Technology, Murthal, Sonepat 131039, Haryana, India*
[b] *Department of Mechanical Engineering, Dr. B. R. Ambedkar National Institute of Technology, Jalandhar 144011, Punjab, India*

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A B S T R A C T

The dynamics of contact between a rigid body rolling on a soft material is quite challenging and interesting. It involves issues of dynamic change in the area of contact and force distribution at the contact interface between the two objects. This dynamics is modeled using multibond graph approach integrated with the Finite Element Method. Bond graph offers the advantage of representation of cause and effect which facilitates the understanding of the complex nature of soft rolling contact. A bond graph model is developed showing interaction between the soft material, rigid body, contact interface, and other subsystems. Stiffness and inertia matrices for the soft material, obtained by using the Finite Element Method, are used as **C** and **I** fields respectively in the multibond graph model. An example of the dynamics of contact of a cylindrical disc, rolling in a controlled manner over a soft material, is considered and explained. The simulation code, in MATLAB, is derived algorithmically from the bond graph model. The model satisfactorily explains issues in the dynamics of rolling contact of rigid objects on soft material interface.

1. Introduction

The first known contact model was developed by Hertz in 1882. He determined that the radius of contact between a spherical surface and a plane is proportional to the normal force raised to the power 1/3. This model is applicable only for small deformation during contact between two linear elastic frictionless surfaces. Later Johnson–Kendall–Roberts [1] included the effect of adhesion in the Hertzian contact model. Their model explained adhesive contact using balance between the stored elastic energy and the loss in surface energy. Xydas and Kao [2] developed a power law theory for soft fingers in contact with a rigid body. They proposed that a nonlinear elastic soft material follows the power law equation

$$a = cN^{\gamma}, \tag{1}$$

where, a is the radius of contact, c is a constant depending on the soft material properties and finger geometry, N is normal force and γ is the constant having value $0 \le \gamma \le 1/3$ depending upon soft material properties. For Hertzian contact $\gamma = 1/3$.

Contact between two objects can be considered either as point contact, or soft contact. The case of point contact is an ideal approximation. While force is transmitted, no moments are considered at contact. In soft contact, forces are distributed over the area of contact, and not just at a point. It is more realistic and also results in development of moments. In applications such as manipulating small rigid objects with fingers, rolling also needs to be considered. Dynamics of rolling soft contact is quite challenging and complex to analyze. The area of contact and force distribution changes during rolling motion.

Efforts have been made to model soft contact using the finite element method (FEM). Xydas et al. [3] modeled soft material using nonlinear FEM. They considered a case when a hemi-spherical soft finger is pressed normally against a rigid plane. Namima et al. [4] modeled and simulated the contact and rolling of an object between two soft fingers using FEM and constrained stabilization method.

* Corresponding author. Tel.: +91 181 2690301x3104.
E-mail addresses: anilnarwal10@yahoo.com (A.K. Narwal), anandvaz@ieee.org (A. Vaz), professorkdg@yahoo.com (K.D. Gupta).

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Vaz and Hirai developed a bond graph model for a ball and socket joint with soft cartilage in between [5]. The functional role of the soft cartilage material was modeled using a nonlinear stiffness field, whose properties could be defined in relation to the ball position with respect to any fixed reference point within the socket. Chang and Cutkosky [6] performed experiments to calculate the kinematic effect of a soft finger on the object during rolling motion. Akella and Cutkosky [7] developed an approximate model for a rigid body rolling over a soft material with hot metal rolling analogy. Anh Ho and Hirai [8] proposed a cantilever model for a sliding fingertip on a rigid plane. But these models do not explain the soft contact rolling dynamics in detail.

It is of interest to understand the effects of change in the area of contact, the deformation of the soft material in keeping with conditions of compatibility, and the distribution of forces as the area of contact changes, during rolling on soft material.

A bond graph model is developed to understand the dynamics of contact interaction during rolling motion of a rigid object on a soft material. As an example, a rigid cylindrical disc is made to roll over a layer of soft material. This example pertains to a planar case in order to keep it computationally simple and economical. However, the approach is applicable to a spatial system as well. The compliant and damping properties of the soft material will affect the rolling motion. These properties are taken into account in the bond graph model. Word bond graph objects for rigid body dynamics and for soft material are developed. The two word bond graph objects are coupled through a contact interface. The structure of the bond graph model is initiated based on flow mapping in which kinematics of the rigid body and that of soft material are represented. The dynamics is subsequently developed on the kinematic structure. The disc is moved in a controlled manner. A proportional derivative (PD) controller decides the force required for the commanded velocity.

The model determines contact area and force distribution while the cylindrical disc rolls over the soft material. Soft contact manipulation can be understood easily using the bond graph approach. The bond graph model is a pictorial representation of system dynamics which depicts transactions of power between interacting subsystems, and the *cause* and *effect* relationships between them. This simplifies the understanding of the dynamics of rolling contact. System differential equations can be derived algorithmically from the bond graph structure. Simulation code is generated directly from the bond graph model. The bond graph model developed in this work is quite general and applicable for a rigid body of any geometry, not just a planar example.

In this paper, FEM formulation for calculating stiffness and mass matrices for the soft material is presented in Section 2. The bond graph model for soft contact is developed in Section 3. Simulation results and experimental validation for soft material deformation and rigid body dynamics are described in Section 4, which is followed by concluding remarks.

2. FEM formulation for calculating stiffness and mass matrices

To investigate the contact phenomenon during rolling, a thin cylindrical disc and a layer of soft material are taken as an example. FEM is used to calculate global stiffness and inertia matrices for the soft material. The soft material layer is meshed into a number of quadrilateral elements as shown in Fig. 1.

The principle of virtual work is used to calculate the global stiffness matrix [9]. Since the contact model is developed in two dimensions, plane stress condition is assumed for the soft material. The width of the soft material is assumed to be negligible. Material matrix [D] for plane stress is given as,

$$D = \frac{E}{1\text{-}\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1\text{-}\nu}{2} \end{bmatrix}, \tag{2}$$

where, E is Young's modulus of elasticity and ν is Poisson's ratio. A general quadrilateral element with local nodes, which are numbered in counterclockwise fashion, and a square shaped master element in coordinates ξ and η, are shown in Fig. 2.

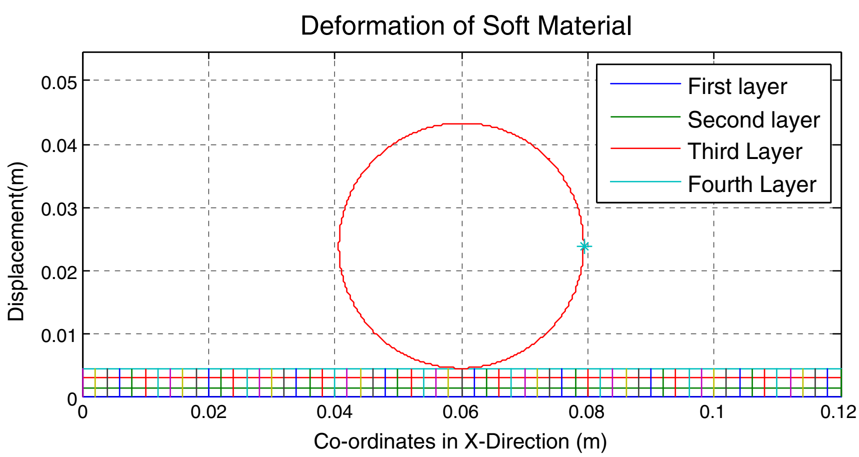

Fig. 1. Thin layer of soft material is meshed into number of quadrilateral elements.