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Curvature based mobility analysis and form closure of smooth planar curves with multiple contacts

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ABSTRACT

This paper presents a simple second-order, curvature based mobility analysis of planar curves in contact. The underlying theory deals with penetration and separation of curves with multiple contacts, based on relative configuration of osculating circles at points of contact for a second-order rotation about each point of the plane. Geometric and analytical treatment of mobility analysis is presented for generic as well as special contact geometries. For objects with a single contact, partitioning of the plane into four types of mobility regions has been shown. Using point based composition operations based on dual-number matrices, analysis has been extended to computationally handle multiple contacts scenario. A novel color coded directed line has been proposed to capture the contact scenario. Multiple contacts mobility is obtained through intersection of the mobility half-spaces. It is derived that mobility region comprises a pair of unbounded or a single bounded convex polygon. The theory has been used for analysis and synthesis of form closure configurations, revolute and prismatic kinematic pairs.

1. Introduction

Mobility is defined as the ability of a rigid body to move in an environment of multiple rigid bodies under mutual constraints. Motion, on the other hand, has been defined in literature as parametric configuration of a rigid body in space. Degrees of freedom (d.o.f.), therefore, can be viewed as the topological dimensionality of this parametric space. Mechanism being a system of rigid bodies under mutual constraints, traditionally, d.o.f. has been studied for mechanisms and the formulation by Grübler for d.o.f. of a mechanism is well known. It is also well known that the equation is not universal; there are several scenarios where the equation fails to predict the right value for d.o.f; this happens when the mechanism has links with special geometry or prismatic joints in special arrangements [1]. The situation gets more complicated when higher pairs are present as contacts. The order of contact which refers to the degree of similarity of the local shapes at the point of contact becomes important. In Fig. 1, (a) has a convex contact, (b) has two circular arcs in contact and (c) has an overlapping curve in contact. It is easy to see that (a) has 1 d.o.f., (b) has 0 d.o.f. and (c) has − 1 d.o.f. Thus, a closer examination of the nature of contact is important to understand the local instantaneous motion of a body as well as the global and gross mobility of a system.

Analyses of mobility and motion of objects in contact are traditionally done using the classical screw theory [2]. The concepts of normal-lines, right-lines, and their configuration in space determine the instantaneous kinematics of a spatial system of rigid bodies [3]. The study of mobility analysis of planar objects in contact dates back to the time of Franz Reuleaux [4,5] who analyzed planar constraints by velocity centers (poles). Although the screw theory is mathematically rigorous, it is not able to interpret the relative motion between rigid bodies in all scenarios [6]. The theory was expanded [6] via the concept of *contrary* and *repelling* screws to accommodate unilateral constraints where violation of contact constraints is possible. Their theories make use of the tangent plane at the point of contact alone to derive the possible motion scenarios. It is therefore not possible to distinguish the cases shown in Fig. 2 wherein (a) infinitesimal motion leads to loss of contact, (b) allows finite sliding with persistent contact and

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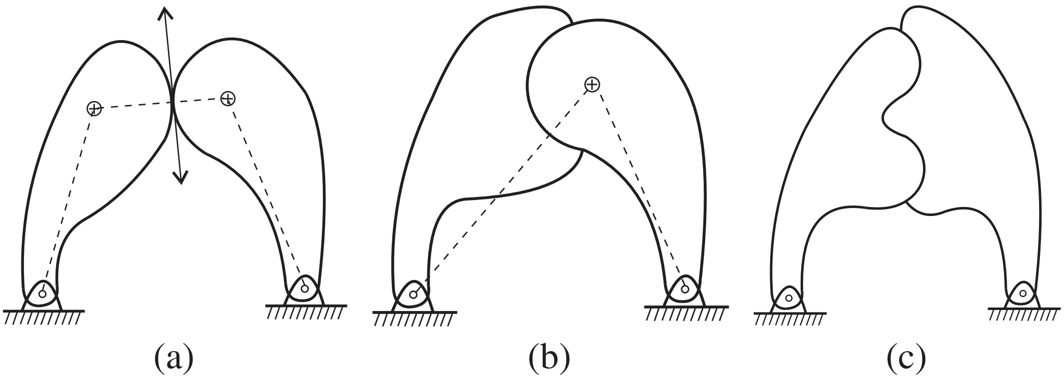

Fig. 1. Contact geometry and d.o.f. of mechanisms. (a) Simple contact single d.o.f., (b) arc contact structure, (c) matching profile over-constrained system.

(c) does not allow any motion. Mobility and contact stability have also been studied in the area of grasping and fixture design within the paradigm of form and force closures [7–10]. However, since the mathematical framework is still the screw theory, the interpretations of a given scenario have the similar limitation as mentioned above.

The limitations of the above methods have led to the emergence of alternate paradigms which consider the higher order local geometric properties of the curves/surfaces at the point of contact [11,12]. In this context, the above methods are categorized as *first-order methods* as they consider only up to the tangent properties. The *second-order methods* incorporate up to the second derivative which is synonymous with the curvature property. A second-order mobility analysis of contacting planar curves in the configuration space characterizing the mobility of the concerned bodies is available in [11,13]. The approach is insightful but it is rather involved for practical use; it can be viewed as an *analysis* tool rather than a *synthesis* one. Second-order form closure analysis using a signed distance function in the Euclidean space has been reported in [12]. Analyses of polyhedral objects in contact [14] and curved objects in contact with polyhedral objects [15] using the "improved screw theory" involving second-order Taylor expansion of the spatial rotation displacement have also been reported in literature.

The work presented here uses geometry of the smooth planar objects directly for the mobility analysis. Small rotation based geometric reasoning as well as Taylor expansion based analytical formulation has been used to derive the differential mobility of smooth contacting curves. A novel contact vector formalism has been proposed, which unifies the problems of mobility analysis, form closure, and synthesis of kinematic pairs.

2. Geometry of contacting objects

The three possible types of contacting geometries are shown in Fig. 3. The three varieties belong to two classes, which we call convex and concave classes. In a convex class contact, the centers of curvature of the two contacting curves are on the either side of the common tangent line, whereas in a concave class contact, they are on the same side of the common tangent line. Therefore a point and tangent to the curve at this point cannot discriminate the three cases. Normal and curvature information are necessary to classify a given contact condition. The second derivatives at the contact give the osculating circles which closely approximate the curves at the point of contact. The fixed and movable curves and their osculating circles are referred to as, *f*-curve, *m*-curve, *f*-circle, and *m*-circle respectively. We refer to the contact normal line as n-line in this paper for brevity. At the point of contact (labeled *C*), a local coordinate system, $x_f y_f$, is embedded in the *f*-curve. Origin of this coordinate system is coincident with the point of contact (*C*). The positive y_f-axis is chosen along the contact normal line in the direction away from the material side of *f*-curve and the direction of positive x_f-axis is chosen so as to make a right handed coordinate frame at the point of contact. The centers of *f*- and *m*-circles are

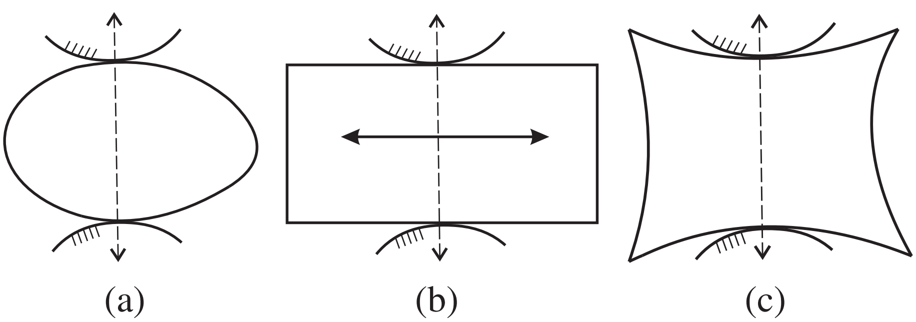

Fig. 2. Instantaneous center analysis cannot distinguish these contacting scenarios.