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Size-dependent elastic properties of micro- and nano-honeycombs

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ABSTRACT

Detailed mathematical derivation and simple closed form results for the size-dependent elastic properties of micro- and nano-sized honeycombs are presented in this paper. The results indicate that at micrometer scale, strain gradient has a dominant effect and at nano-meter scale, surface elasticity dominates the effect on the honeycomb elastic properties. The in-plane elastic properties of a nano- or micro-honeycomb could be controlled to vary over a range of around 10% by adjusting the initial stress in the cell walls by applying an electric potential. In addition, the bending and shear rigidities of some commonly used micro- and nano-structural elements have been obtained and presented in this paper, which could be of important applications in the design of MEMS and NEMS.

1. Introduction

With the rapid development of technology, it is now possible to produce regular micro- and nano-sized honeycombs (Nishihara et al., 2005). Such micro- and nano-cellular materials have found many applications in MEMS and NEMS. For example, they can be used as the actuator material of nano-sensors (Weissmuller et al., 2003; Kramer et al., 2004), or for sorption and separation (Stein, 2003; Davis, 2002), drug delivery, energy storage and fuel cell technology (Lu and Zhao, 2004). In all these applications, the predictability and reliability of the mechanical properties, such as the strength, stiffness and deformation behaviour, are extremely critical in ensuring the function of the MEMS or NEMS. Although the mechanics theories of macro-honeycombs have well been established (e.g. Gibson et al., 1982; Gibson and Ashby, 1997; Zhu and Mills, 2000; Zhu et al., 2001, 2006), their results may not apply to their micro- and nano-sized counterparts. Nano- and micro-honeycombs are made of plates whose thickness is at nano- or micro-meter scale. It has generally been recognised that at the micrometer scale, the strain gradient effect plays an important role in the mechanical behaviour (Toupin, 1962; Mindlin and Tiersten, 1962; Mindlin, 1963; Fleck and Hutchinson, 1993; Aifantis, 1999; Nix and Gao, 1998; Gao et al., 1999; Anthoine, 2000; Lubarda and Markenscoff, 2000; Haque and Saif, 2003; Lam et al., 2003; McFarland and Colton, 2005; Zhu and Karihaloo, 2008). The classical continuum mechanics theory cannot be used to interpret this size-dependent effect as it does not contain any material length scale parameter. The classical couple-stress elasticity theory (Toupin, 1962; Mindlin and Tiersten, 1962; Mindlin, 1963; Anthoine, 2000; Lubarda and Markenscoff, 2000) has been developed to describe the size-dependent effect. However, as it involves two separate material length scale parameters (Toupin, 1962; Mindlin and Tiersten, 1962; Mindlin, 1963; Anthoine, 2000; Lubarda and Markenscoff, 2000; Papargyri-Beskou et al., 2003; Vardoulakis and Giannakopoulos, 2006), it is a very difficult task to experimentally determine the micro-structural material length scale parameters (Yang and Lakes, 1982; Yang et al., 2002). Yang et al. (2002) have recently developed a

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couple-stress theory, in which the strain energy has been shown to be a quadratic function of the strains (which are symmetric) and the symmetric curvatures and only a single material length scale parameter is involved. It is much easier to deal with a single material length scale parameter in either theoretical analysis or experimental measurement.

At nano-meter scale, both the surface elasticity (Miller and Shenoy, 2000; Duan et al., 2005; Wang et al., 2006) and initial stresses (Zhu, 2008; Zhu et al., 2009) can greatly affect the mechanical properties of structural elements. Miller and Shenoy's (2000) and Sun et al.'s (2007) atomistic simulations suggested that for metallic structural elements with a size of a few nano-meters, strain gradient effect is irrelevant, and surface elasticity or surface energy dominates the influence on the mechanical properties.

In this paper we will start with the general expressions of the five independent elastic constants of regular honeycombs, and then briefly introduce the results of macro-sized honeycombs. After that, we will present the size-dependent results of those five elastic constants for honeycombs with uniform cell walls of thickness at micro- and nano-meter scales.

2. General expressions of the five independent elastic constants

A regular hexagonal honeycomb, as shown in Fig. 1a, has a plane of isotropy and has only five independent elastic constants (Nye, 1985; Gibson and Ashby, 1997; Kim and Christensen, 2000). Under small deformation, the effective elastic stress and strain in a regular honeycomb are related by (Gibson and Ashby, 1997)

$$\begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \gamma_{23} \\ \gamma_{31} \\ \gamma_{12} \end{pmatrix} = \varepsilon_i = S_{ij}\sigma_j = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{12}}{E_1} & -\frac{\nu_{31}}{E_3} & 0 & 0 & 0 \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_1} & -\frac{\nu_{31}}{E_3} & 0 & 0 & 0 \\ -\frac{\nu_{13}}{E_1} & -\frac{\nu_{13}}{E_1} & \frac{1}{E_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{31}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{31}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_{23} \\ \sigma_{31} \\ \sigma_{12} \end{pmatrix} \tag{1}$$

where $\nu_{ij} = -\varepsilon_j/\varepsilon_i$. The in-plane (i.e. x–y plane) isotropy requires

$$G_{12} = \frac{E_1}{2(1+\nu_{12})} \tag{2}$$

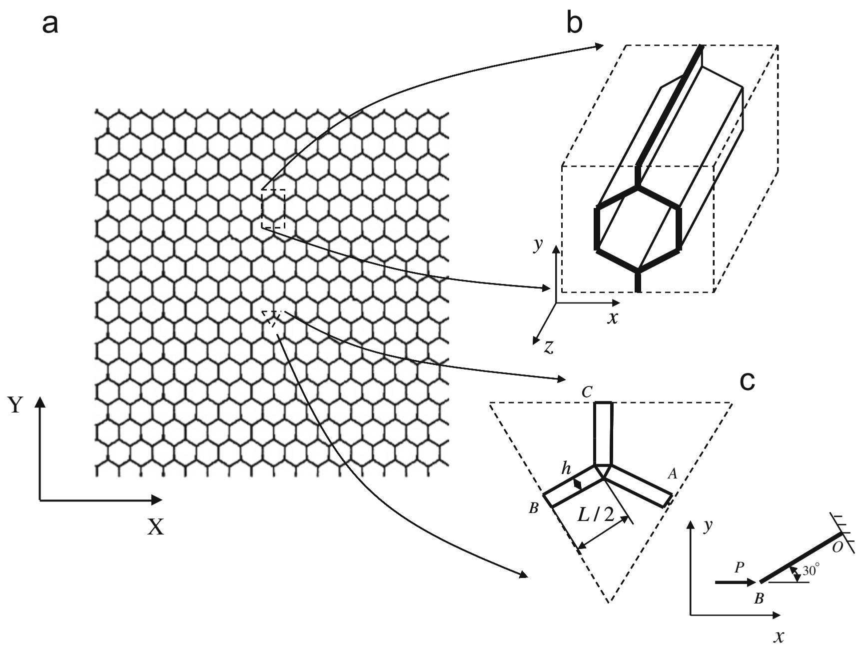

Fig. 1. A regular hexagonal honeycomb with cell walls of uniform thickness.