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On the mechanical properties of hierarchical lattices

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Hierarchical lattices are made of finer lattices in successive smaller scales. This paper analytically studies the effect of hierarchy on the stiffness and strength of self-similar and hybrid type lattices, made by combining two distinct variants of topologies, governed by the bending and stretching dominated architectures. Scaling argument and physical reasoning are used to explain the behaviour of these lattices. The results show that the in-plane stiffness and the elastic buckling strength of the bending–bending lattices progressively improve with hierarchy; in contrast, only the buckling strength improves substantially for the stretching–stretching lattices, while the stiffness decreases. Low density bending–stretching lattices are unique with a significant improvement in stiffness, buckling, plastic collapse or crushing strength with hierarchy, whereas the stretching–bending lattices exhibit flexibility with lower strength. Despite no gain in stiffness, substantial gain in out-of-plane compressive strength is obtained with hierarchy because of the enhanced elastic and plastic buckling strength. Thus the advantage of combining lattices at multiple length scales provides a wide spectrum of choices for tailoring the properties for target applications including high performance core material, energy absorption or packaging.

1. Introduction

Natural materials including wood, bone are stiff, strong and tough materials with a unique property that they are porous, hence, lightweight. A closer look at their microstructure reveals that they are made of hierarchical cellular materials (Lakes, 1993; Fratzl and Weinkamer, 2007), i.e., a cellular architecture that is made of finer cells in successive smaller scales. Thus they exhibit structures at multiple length scales. This hierarchical architecture provides the necessary stiffness, strength and crushing toughness in the lightweight natural materials. A good example of a man-made hierarchical structure is the Eiffel Tower. The man-made analog of natural cellular materials, honeycombs and foams, are collectively known as *lattice materials.* These class of materials are made of interconnected slender rods, beams or plates that lead to a porous microstructure. Due to the excellent specific stiffness and specific strength (stiffness and strength to weight ratio), apart from energy absorbers and packaging applications, these materials have been extensively used as sandwich core materials in aerospace and marine industries and, are now being used in infrastructure and sports goods industries. Their multi-functional properties including sound absorption, thermal insulation and vibration control have also attracted significant interest (Evans et al., 2001).

With an aim to produce low density high performance cellular materials, the research focus broadly includes the development of new topologies, lattices made of new materials and performance optimisation. For example, study of properties of Kagome-like cellular solids (Hyun and Torquato, 2002), performance study of metallic sandwich panels with metallic textile cores (Zok et al., 2003)

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and pyramidal truss cores (Zok et al., 2004); performance optimisation of honeycomb cores made of novel reinforced thermoplastic materials (Banerjee et al., 2010) and corrugated core made out of wood veneer sheets (Banerjee and Bhattacharyya, 2011). In parallel, researchers have developed analytical tools correlating the macroscopic mechanical properties with the topology, cell geometry and mechanical properties of the cell wall material (Gibson et al., 1982; Warren and Kraynik, 1987; Gulati, 1975). Vast literature on cellular materials can be found in the book by Gibson and Ashby (1997), review papers by Christensen (2000) and Fleck et al. (2010), and the references thereof.

Bio-mimetic approach to material design has inspired the research on hierarchical cellular materials. Bhat et al. (1989) showed experimentally that the compressive strength of sandwich panels with second order honeycomb cores is about six times greater as compared to an equal weight sandwich panel with a first order core. According to the best of author's knowledge, the pioneering theoretical work in this area has been done by Lakes (1993). He developed expressions for the stiffness and strength of hierarchical cellular materials and, experimentally observed significant improvement (more than 3 orders of magnitude) in the out-of-plane compressive strength for the hierarchical hexagonal honeycombs. Recently, failure mechanisms of second order corrugated sandwich core materials under transverse loading have been investigated both theoretically and experimentally by Kooistra et al. (2007) and Kazemahvazi and Zenkert (2009a,b). Fan et al. (2008) showed theoretically that substantial enhancement in the mechanical properties can be achieved with the second order triangular and hexagonal lattices made of sandwiches. In another study, Chen and Pugno (2012) analysed the in-plane elastic buckling of hierarchical orthotropic honeycombs and studied the progressive failure. Vigliotti and Pasini (2013) used a multiscale procedure for analysing the mechanical properties of various hierarchical lattices up to the third order. Sen et al. (2011) showed that the size-dependent nano-sized honeycombs are tougher as compared to larger size honeycombs. Torrents et al. (2012) manufactured and tested third order metallic lattices and showed one order of magnitude increase in the stiffness and strength. Recent advancement of the rapid prototyping techniques such as additive manufacturing (Stamfl et al., 2004; Ramirez et al., 2011) has also motivated this research. For example, Rayneau-Kirkhope et al. (2012) used fractal approach for designing ultralight beam from hollow tubes and manufactured a hierarchical beam using rapid prototyping techniques. Therefore, hierarchical cellular materials have the potential of improved stiffness and strength at low density and thus, offers a route of producing ultralight yet high performance materials.

This work focuses on the mechanical behaviour of lattice materials with structural hierarchy. A solid material can be viewed as a continuum at macroscale, it is a zeroth order cellular material; traditional lattice made of triangular or hexagonal cells is of rank 1 and, in a lattice of rank 2, each cell wall is made of smaller cells at a finer scale and so on. Nodal connectivity, i.e., how many cell walls connected at a node plays an important role in determining the deformation mechanisms of the cell walls under load, and, in turn, the macroscopic properties of the lattice. For example, the deformation mechanism of a truss-like triangular lattice with a nodal connectivity of six is stretching dominated, whereas a hexagonal lattice with a nodal connectivity of three is primarily bending dominated (Deshpande et al., 2001). Thus, these two topologies represent two distinct types of lattices. A generic route of making hierarchical lattices would be to combine these two variants. Combinations of these two lead to four types of hierarchical lattices. Two types of hierarchies are made of similar cells at multiple scales, so self-similar: (a) *stretching-stretching* and (b) *bending-bending*. Third and fourth possible types of hierarchies are hybrids, namely, (c) *bending-stretching* and (d) *stretching-bending*. For instance, a self-similar stretching-stretching lattice of rank 2 is a triangular lattice, with the cell walls being made of smaller triangular cells at a finer scale; in contrast, a hybrid bending-stretching lattice of rank 2 is a hexagonal lattice, with the cell walls are made of smaller triangular cells at a finer scale. Fig. 1 shows these four type hierarchies.

In this paper, we explore the effects of hierarchy on the mechanical properties of the aforementioned lattices. In particular, we address the following three issues. Firstly, we quantify the differences between the mechanical properties of the stretching and the bending-dominated architectures (self-similar types) with hierarchy. It is known that a triangular lattice exhibits stretching dominated behaviour and it is the stiffest (Torquato et al., 1998), whereas bending dominated hexagonal lattice is a lot less stiffer than a triangular lattice. So the question is what happens to the stiffness of a stretching dominated lattice with hierarchy? What happens to the stiffness of a multiscale bending dominated lattice? How does the failure

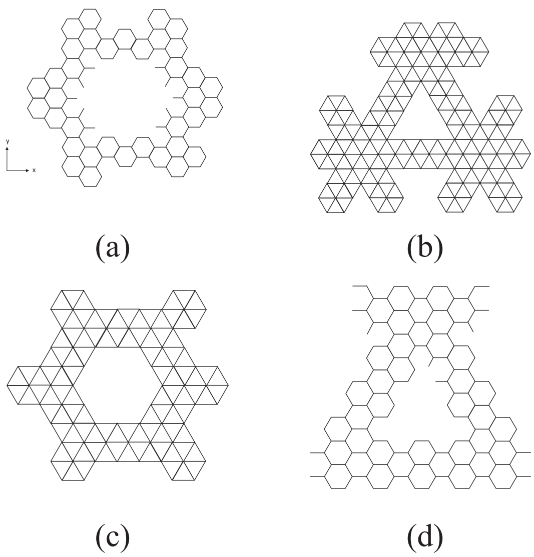

Fig. 1. Hierarchical lattices of rank 2. Self-similar types: (a) A bending-bending lattice: hexagonal lattice made of smaller hexagonal cells at a finer scale. (b) A stretching-stretching lattice: triangular lattice made of finer triangles. Hybrid types: (c) A bending-stretching hierarchy with a hexagonal lattice made of triangles at a finer scale. (d) A stretching-bending lattice, with a triangular lattice made of finer hexagonal cells.