Mechanics of Materials 52 (2012) 12–27

Contents lists available at SciVerse ScienceDirect

Mechanics of Materials

journal homepage: www.elsevier.com/locate/mechmat

Numerical modelling of the fracture behaviour of brittle materials reinforced with unidirectional or randomly distributed fibres

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ARTICLE INFO

Article history:
Received 11 April 2011
Received in revised form 6 April 2012
Available online 25 April 2012

Keywords:
Brittle materials
Fracture
Fibre-reinforced composite
Micromechanical model
Fibre debonding

ABSTRACT

The use of reinforcing fibres has shown to be an effective, simple and economic way to enhance the mechanical characteristics of brittle materials; in particular tensile strength, fracture and fatigue resistance, wear resistance and durability are usually noticeably higher in fibre-reinforced materials (FRC) with respect to unreinforced ones. For the above mentioned reasons composite materials today can replace or compliment other traditional structural materials.

On the other hand the extensive use of brittle matrix composite materials requires appropriate computational models to describe, with adequate accuracy, their mechanical behaviour. In the present paper a mechanical-based computational model for the description of the macroscopic behaviour of such a class of materials, composed by a matrix phase and a fibre-reinforcing phase, is formulated. By considering a micromechanical-based model, the macro constitutive equations of unidirectional or randomly distributed fibres reinforced materials are obtained by taking into account the possibility of crack formation and propagation in the matrix as well as fibre debonding and breaking. The developed computational model is finally used in some numerical simulations in order to outline its reliability in the assessment of both the fibre–matrix interaction phenomenon as well as the fracture failure prediction capability in brittle matrix FRC materials.

1. Introduction

Several well-known drawbacks of brittle or quasi-brittle materials – such as the low tensile strength, low fracture and fatigue resistance, scarce wear resistance and durability under repeated loading, etc. – have been observed to be eliminated, or at least mitigated, by the use of reinforcing fibres. The use of fibres dispersed in a material to improve its mechanical properties, has been a typical technique used in practical applications since ancient ages.

The appropriate use of reinforcing fibres in brittle materials has shown to be an effective, simple and economic way to enhance their mechanical characteristics, allowing to get materials which are competitive with more technologically advanced ones. For the above mentioned reasons the use of composite materials in many application fields has known an increasing interest in the last decades allowing to replace or compliment other traditional structural materials. In particular FRC are usually characterised by high strength, fracture and fatigue resistance, high wear resistance, durability performance, high damping capability, low thermal coefficient, and so on.

The extensive use of composite materials has determined the necessity to describe, with an appropriate accuracy from the engineering point of view, their overall mechanical behaviour to correctly assess their safety level in the design of structural components. In order to obtain suitable mechanical model for such materials, various approaches have been developed such as micromechanical models (physically based approach, Hori and Nemat-Nasser, 1999; Kalamkarov and Liu, 1998; Kalamkarov et al., 1998), homogenisation models (mathematically

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0167-6636/$ - see front matter
http://dx.doi.org/10.1016/j.mechmat.2012.04.008

Nomenclature

A_f	cross section area of the fibres
$\mathbf{b}$, $\hat{\mathbf{t}}$	body force vector field and traction field on the loaded boundary, respectively
$\mathbf{B}$, $\mathbf{B}(\mathbf{x})$	generic compatibility matrix of the finite element and its corresponding value at the location $\mathbf{x}$ of the finite element, respectively
$\mathbf{B}^+(\mathbf{x}) = \sum_{i\in\Omega_e^+} B_i(\mathbf{x})$	sum of the compatibility matrices' values evaluated at the location $\mathbf{x} \in \Omega_e^+$
$\mathbf{C}'_m$, $\mathbf{C}'_f$, $\mathbf{C}'_{eq}$	tangent elastic tensor of the matrix, of the fibres and of the homogenised composite, respectively
d, D	characteristic microscopic and macroscopic length, respectively
E_f	young modulus of the fibres
E_c, E_{cx}, E_{cy}	young modulus of the composite material for the isotropic case and along the directions x and y for generic anisotropic cases, respectively
$f_{t,f}$	fibre tensile strength
G_f	fracture energy of the matrix per unit surface crack area
$H(\mathbf{x})$	heaviside jump function
$\mathbf{i}$, $\mathbf{j}$	unit vectors normal and parallel to the crack direction, respectively
$\mathbf{k}$	unit vector parallel to the generic fibre axis
$2L_f$	length of the fibres
$\mathbf{N}$, $\mathbf{N}(\mathbf{x})$	generic shape functions and shape functions matrix evaluated at the location $\mathbf{x}$, respectively in a FE
$\mathbf{N}^+(\mathbf{x}) = \sum_{i\in\Omega_e^+} N_i(\mathbf{x})$	sum of the shape functions evaluated at the location $\mathbf{x} \in \Omega_e^+$
$p_\varphi(\varphi)$, $p_\theta(\theta)$	probability density functions of the angles φ and θ, respectively
$\mathbf{Q}$	element nodal discontinuity matrix
r_c, $2r$	crack surface roughness and diameter of the fibres, respectively
RVE	representative volume element
S	discontinuity locus in a cracked solid
$s(\varepsilon_f^m)$, $s(\overline{\varepsilon_f^m})$	sliding function such that: $[[\varepsilon_{f-m}]] = \varepsilon_f^m \cdot [1 - s(\varepsilon_f^m)]$ and its mean value along a single fibre
$\delta\mathbf{u}$, $\delta\mathbf{w}$	kinematically admissible displacement fields
u_c, v_c	normal and parallel component to the crack of the relative displacement vector across the crack, respectively
u_0	minimum opening crack displacement corresponding to the crack formation
V, V_m, V_f	volume of the composite, volume of the matrix phase and volume of the fibre fraction present in the REV, respectively
$\mathbf{w}_c = \mathbf{u}_c + \mathbf{v}_c = u_c\mathbf{i} + v_c\mathbf{j}$	relative displacement vector across the crack faces
$\mathbf{w}$, $\mathbf{w}_n$	displacement jump vector and its nodal counterpart, respectively
$\mathbf{x}$	position vector
$\boldsymbol{\delta}(\mathbf{x})$, $\bar{\boldsymbol{\delta}}(\mathbf{x})$	displacement vector and its continuous part, respectively in a generic solid or in a finite element
$[[\boldsymbol{\delta}(\mathbf{x})]] = \mathbf{w}(\mathbf{x})$	displacement jump vector in a generic point $\mathbf{x} \in S$
$\boldsymbol{\delta}_d(\mathbf{x})$	discontinuous part of the displacement field
δ_s	dirac delta function located in S
δ_φ, δ_θ	variance of the probability density functions $p_\varphi(\varphi)$ and $p_\theta(\theta)$, respectively
$\boldsymbol{\varepsilon}(\mathbf{x})$	strain tensor evaluated at the location $\mathbf{x}$
$\boldsymbol{\varepsilon}^b(\mathbf{x})$, $\boldsymbol{\varepsilon}^u(\mathbf{x})$	bounded and unbounded part of the strain tensor evaluated at the location $\mathbf{x}$, respectively
$\boldsymbol{\varepsilon}$, $\boldsymbol{\sigma}$	strain and stress tensors, respectively
$\delta\boldsymbol{\varepsilon}$	strain field corresponding to a kinematically admissible displacement fields
$[\varepsilon_{f\text{-}m}]]$	strain jump between the fibre and the matrix (parallel to fibre axis) in the case of imperfect bond
ε_f, ε_f^m	uniaxial fibre strain and uniaxial matrix strain measured at the location and in the fibre direction, respectively
$\overline{\varepsilon_f^m}$	matrix mean strain measured in the fibre direction
φ, θ	angles indicating the fibre orientation in the 3D space
$\bar{\varphi}$, $\bar{\theta}$	mean values of the probability density functions $p_\varphi(\varphi)$ and $p_\theta(\theta)$, respectively
$\mu = V_m/V$, $\eta = V_f/V$	REV matrix volume fraction and fibre volume fraction, respectively
$\boldsymbol{\sigma}(\mathbf{x})$	stress tensor evaluated at the location $\mathbf{x}$
$\sigma_c(u)$, $\tau_c(u)$	stress-crack opening displacement and shear stress-crack opening displacement relationship, respectively
τ_{au}, τ_{fu}	maximum fibre–matrix interface shear stress and friction fibre–matrix interface shear stress, respectively
$\Gamma = \Gamma_t \cup \Gamma_u$	boundary of the solid
Γ_t, Γ_u	portion of the boundary on which tractions and displacements are prescribed, respectively

based approach, Balendran and Nemat-Nasser, 1995; Hassani and Hinton, 1998), etc.

Moreover brittle or quasi-brittle materials (often used as matrix materials in fibre-reinforced composites) frequently present cracks, which mathematically correspond to a severe strain localisation phenomenon, leading to collapse due to fracture propagation; it is well known as the numerical simulation of such a class of problems presents several difficulties (computational instabilities, divergence or non-uniqueness of the solution, etc.) due to the discontinuous displacement field which develops in highly strained narrow zones.

Several models have been developed to solve the mechanical problems stated above; in this context the classical smeared approaches (suffering by mesh dependence, Bolzon et al., 1997) – eventually improved with some specific strategies and corrections such as remeshing or mesh adaptivity (Belytschko and Black, 1999; Bouchard et al., 2000; Comi and Perego, 2004; Möes et al., 1999; Rashid, 1998) – finite elements enrichment (Dolbow