Available online at www.sciencedirect.com

ScienceDirect

Journal of Terramechanics 58 (2015) 11–25

Journal of Terramechanics

www.elsevier.com/locate/jterra

Statistical modeling and comparison with experimental data of tire–soil interaction for combined longitudinal and lateral slip

Jonah H. Lee

Department of Mechanical Engineering, University of Alaska Fairbanks, Fairbanks, AK 99775-5905, United States

Received 8 July 2014; received in revised form 24 November 2014; accepted 26 December 2014
Available online 19 January 2015

Abstract

The interaction of a tire with a soft terrain has multiple sources of uncertainties such as the mechanical properties of the terrain, and the interfacial properties between the tire and the terrain. These uncertainties are best characterized using statistical methods such as the development of stochastic models of tire–soil interaction. The quality of the models can be assessed via statistical validation measures or metrics. Although validation of stochastic tire–soil interaction models has recently been reported with good results, it involves longitudinal slip only without considering lateral slip which can occur simultaneously with longitudinal motion. This paper presents results of the validation of a simple stochastic tire–soil interaction model for the more complicated case of combined slip. The statistical methods used for validation include the development of a Gaussian process metamodel, the calibration of model parameters using the approach of the maximum likelihood estimate in conjunction with new test data. The validation of the calibrated model, when compared with test data, is obtained using four validation metrics with good results.

Keywords: Soil; Drucker–Prager; Slip angle; Validation metrics; Calibration; Bayesian; Metamodel; Stochastic; Gaussian process; Uncertainties

1. Introduction

Although tire–soil interaction involving longitudinal slip has been much studied in the past, relatively less studies have been conducted for combined longitudinal and lateral slip which include, as examples, work reported in Krick (1973), Karafiath (1986), Crolla and El-Razaz (1987), Armbruster and Kutzbach (1991), and Muro and O'Brien (2004). Most of these studies, however, are for single-tire, soil-bin type of tests under more controlled laboratory conditions assuming homogeneous soil conditions, as opposed to our focus for the more challenging situation of a full vehicle maneuvered by a human driver in the field with various uncertainties as a result of the variations of the properties of soil, their interaction with the tire, and the terrain profile.

One way to tackle the uncertainties is to use stochastic models as opposed to deterministic ones. The work in Li and Sandu (2007) is one of the few studies in the stochastic modeling of tire–soil interaction; however, no validation was attempted.

Recently, as part of systematic and comprehensive studies of the uncertainties of tire–terrain interaction, statistical validations for stochastic models have been conducted where new test data were obtained using an instrumented test vehicle (Lee et al., 2010, 2012). The models that have been validated – with room for improvement – include: indentation model for snow (Lee and Huang, 2012), tire–snow interaction model as a function of longitudinal slip (Lee, 2013), tire–snow interaction in the time domain under combined slip in Lee and Huang (2014), and tire–soil interaction in the time domain for longitudinal motion in Lee and Gard (2014).

E-mail address: jonah.lee@alaska.edu

http://dx.doi.org/10.1016/j.jterra.2014.12.005

Nomenclature

$\tau_{\text{mod}}, \tau_{zx}, \tau_{zy}$	total, longitudinal, and lateral friction-limited shear stress at tire–snow interface
α	slip angle
$\bar{z}$	distance from the centroid of the lateral contact area to tire center
$\bar{\sigma}$	von Mises stress
β	friction angle for Drucker–Prager criterion
ϵ^p	volumetric plastic strain
$\mathbf{x}_{\text{calibrated}}$	calibrated parameters
μ	coefficient of Coulomb friction
ω	angular velocity
ϕ	friction angle for Mohr–Coulomb criterion
σ_n	normal stress on tire
σ_y	normal stress on tire in the lateral direction
τ_Σ	shear stress at tire–snow interface
$\mathbf{x}$	parameters in the statistical model
θ, θ_0	angular position, and exit angle of tire
A_y	contact area in the lateral direction
b	width of tire
c	cohesion for Mohr–Coulomb criterion
c_1	hardening constant of soil plasticity model
D	diameter of tire
$E(t)$	mean error between model and test as a function of time
$f(\mathbf{x})$	target variables of the statistical model as a function of parameters **x**
F_x, F_y, F_z	longitudinal drawbar pull, lateral drawbar pull (force), normal force on tire
f_{rr}	rolling resistance of tire itself
F_{zx}, F_{zy}	longitudinal, lateral traction
i_x	longitudinal slip
j_x, j_y, j_Σ	longitudinal, lateral, and total shear displacement
j_{y0}	maximum lateral shear displacement
K_{shear}	shear stress–shear displacement modulus
M_x	overturning moment
M_y	torque on tire
p	hydrostatic pressure
p_a, p_d	yield surface cap location, and cohesion for Drucker–Prager criterion
r	tire radius
R_x, R_y	longitudinal, and lateral motion resistance
v_x, v_y	longitudinal, and lateral velocity
Y	test data in the statistical model
z, z_0	sinkage, and maximum sinkage of tire

It should be noted that, *validation* used in this paper follows the recognized definition in the literature (ANSI/ASME V& V 10-2006, 2006) as: 'the process of determining the degree to which a model is an accurate representation of the real world from the perspective of the intended uses of the model'. How well a model is validated thus depends on appropriate measures and metrics for the intended use of the model. In addition, for the validation of *stochastic* models, two types of models are involved – a deterministic vehicle–terrain model and a statistical model. This is different from the traditional approach of assessing the performance of deterministic non-stochastic models.

The statistical validations of the above-mentioned models are based upon a flexible statistical framework which can be tailored for specific applications. The framework has several components: the building of a stochastic metamodel as a surrogate of the physical model, calibration of model parameters using the statistical method of maximum likelihood estimate, prediction and validation using the metamodel, calibrated parameters, test data and validation metrics. The quality of the physical and statistical models is assessed using several validation metrics, in conjunction with test data, such that a decision can be made regarding the need, if any, to improve the physical model, and/or the statistical model, and/or the test.

The work reported in Lee and Gard (2014), however, involves longitudinal motion and slip only without considering the more complicated problem of combined slip. The purpose of this paper is then to validate statistically a simple tire–soil interaction model under combined slip. The paper is organized as follows. Section 2 summarizes the essential ingredients of the tire–soil interaction model. The statistical methods are summarized in Section 3. Section 4 discusses the experimental procedures. Section 5 presents comprehensive results that include vehicle states and soil properties, results of calibration, comparison with test data using validation metrics. The paper closes with discussion and conclusions in Section 6.

2. Tire–soil interaction

The tire–soil interaction model is an extension of the one for longitudinal slip in Lee and Gard (2014). The inclusion of the lateral slip uses the same approach in Lee and Huang (2014) for tire–snow interaction. Following (Lee and Gard, 2014), the deformation of the tire is assumed to be negligible, i.e., this a rigid-tire model. The model uses two submodels: the soil material model, and the soil indentation model. The definitions and essential components of the model are given below; more details can be found in Lee and Huang (2014) and Lee and Gard (2014).

2.1. Material model

A simple Drucker–Prager model (Lee and Gard, 2014) can be expressed as: