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The use of central reflection in the formulation of unit cells for micromechanical FEA

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ABSTRACT

Central reflection as a type of symmetry has been exploited in the context of structural analysis for the first time in this paper, especially in terms of applications to the formulation of unit cells for micromechanical FE analysis of materials of periodic microstructures. It reduces the size of the unit cell to be analysed without compromising useful features of the unit cells obtained purely from periodic conditions, *viz.* the unit cell can still be analysed with a single set of boundary conditions under all loading conditions and any combined loading condition. Such a feature cannot be preserved if a conventional plane reflectional symmetry or a rotational symmetry has been used in order to reduce the size of the unit cell to be analysed. Most of existing unit cells obtained purely from periodic conditions can benefit from this further symmetry. Boundary conditions for unit cells of reduced sizes resulting from the use of the centrally reflectional symmetry have been derived rationally in this paper. They have been validated systematically through examples of applications.

1. Introduction

Symmetry as a geometric property is a well understood topic (Hamermesh, 1962). Extensive use has been found in classification of crystals (Nye, 1985). Theoretically, there are three generic types of symmetries: translations, reflections and rotations. All of them have been used to different extents in structural analyses, with reflections being the most attended type to such an extent that they are sometimes perceived as the symmetry. Such a perception is transpired through the fact that it is the only type of symmetry that has been incorporated in most commercial FE codes. To implement other types, such as translations and rotations, the user will have to improvise, which may not be always possible, e.g. when the facility of equation boundary conditions or multiple point constraints is not available in the code. Fortunately, many mainstream commercial FE codes do have such a facility, such as ABAQUS.

The three generic types of symmetries have also been utilised extensively in formulating unit cells for micromechanical analyses of materials of regular microstructures. Of them, translations result in the commonly called periodic conditions (Suquet, 1987). The benefits of using translational symmetries alone have been elaborated in reasonable length in Li (1999, 2001, 2008), Li and Wongsto (2004) and Li et al. (2011a,b). The advantage of a unit cell so formulated is that a single set of boundary conditions applies for any loading condition in terms of a single macroscopic stress component or a combination of several of them. The existence of further symmetries, such as reflections and rotations, can be used to reduce the size of the unit cell, as has been illustrated in analyses of unidirectional (UD) composites (Li, 1999) and more recently plain weave textile composites (Li et al., 2011a). However, the use of reflectional and rotational symmetries tends to restrict the applicability in terms of combined loading

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conditions. Also, different sets of boundary conditions have to be imposed when the unit cell is under different loading conditions. While a single set of boundary conditions offers convenience in unit cell formulation and applications, reduced unit cell size is computationally attractive, especially when dealing with complicated unit cells, e.g. for textile composites where high demand on computing power often arises (Li et al., 2011b). A compromise, while highly desirable, has not been available.

Mathematically, these three types of symmetries are mutually independent and collectively comprehensive so that any other type of symmetry, such as reflection about the central point, reflection about a central line, etc., can be obtained as a simple combination of some of these generic symmetries and is hence not independent. The lack of independence of such derived symmetries has probably deterred users from exploiting them. As a result, they have never been employed in structural analyses to the best of the authors' knowledge, except very special ones, such as axisymmetry and spherical symmetry.

A central reflection is a combination of a plane reflection and a 180° rotation. It is a common symmetry widely available in reality. In many cases, this symmetry coexists with many others, e.g. the surface pattern on a football, and hence it does not easily catch ones eyes. The existence of such a geometric symmetry is not really exciting. An interesting mechanical feature of this symmetry is that any stress or strain state possesses this symmetry. As a result, the mechanical outcome of this symmetry will be shown to be rather significant. Taking advantage of this when it is available in a unit cell, it will produce a set of boundary conditions which is applicable to all loading cases as well as any combination of macroscopic stresses, while halving the size of the unit cell. Neither a plane reflection nor a rotation alone delivers such a property. This useful feature distinguishes this particular symmetry from plane reflections or rotations, even though there is a generic relationship in between in terms of geometric symmetry considerations. This states the fact that symmetry is not a pure geometric issue when it is employed in mechanics, where mechanical considerations crossbreed with geometric ones to generate a more sophisticated study.

In this paper, the use of central reflection will be considered and the outcomes are of significant interest to a wide community of structural analysis even though the presentation of this paper will be mostly confined to the context of unit cells for micromechanical FE analyses. As this symmetry and its useful features do not seem to have been paid any attention in the literature, hitherto, the demand of a properly documented account in the open literature justifies the present paper.

2. Central reflection

A centrally reflectional symmetry is often observed in popular objects. However, patterns and shapes possessing this symmetry often show other symmetries, such as reflections and rotations, making the recognition of central reflection either redundant or unobvious. A simple but distinctive shape of central reflection is a triclinic crystal

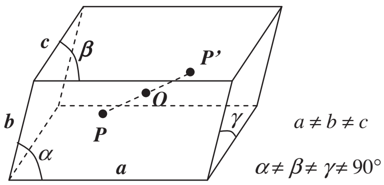

Fig. 1. A triclinic crystal.

(Nye, 1985), Fig. 1, which is a hexahedron of three parallel, unnecessarily orthogonal, pairs of faces and three independent side lengths. In this geometry, central reflection is the only symmetry available.

The analytical description of the central reflection, denoted as *CR* hereafter in this paper, can be given as a mapping

$$CR : P \rightarrow P'. \tag{1}$$

where P is an arbitrary point in the triclinic body as origin and P' as its image under the mapping. Assuming the coordinates of P and P' are (x,y,z) and (x',y',z'), respectively, one has

$$(x' - x_0, y' - y_0, z' - z_0) = -(x - x_0, y - y_0, z - z_0)$$

or

$$(x', y', z') = (2x_0 - x, 2y_0 - y, 2z_0 - z), \tag{2}$$

where (x_0, y_0, z_0) are the coordinates of the centre O for the central reflection. If the three generic types of symmetries are conventionally denoted as (Hamermesh, 1962)

(1) T_A^{Δ}: Translation along axis A by a distance of Δ,
(2) Σ_A^P: Reflection about a plane perpendicular to axis A at point P, and
(3) C_A^n: Rotation of 360°/n about axis A.

a central reflection is then a combination of Σ_A^O and C_A^2 where A is an arbitrary axis passing through the centre O. As a combination of two generic symmetries, *CR* is not independent of the three generic symmetries mathematically. However, the interesting and useful characteristics of this symmetry are its mechanical consequence, which will be elaborated in the next section.

3. Nature of symmetries

Mechanical applications of geometric symmetries are complicated by the existence of two different natures of symmetries, *viz.* symmetry and antisymmetry, which are related to applied loads, as well as the internal stresses and strains, in presence of geometric symmetry of the structure. Loading conditions for micromechanical analyses of unit cells are typically expressed in terms of macroscopic stresses or strains (average stresses or strains). They preserve symmetric nature under translational symmetry transformations. This is responsible for the fact that a single set of boundary conditions are applicable for all loading conditions if only translational symmetries are used.

It is well known that reflectional and rotational symmetries only preserve the senses of some stress and strain