Available online at www.sciencedirect.com
ScienceDirect
Journal of the Mechanics and Physics of Solids 56 (2008) 1905–1915
JOURNAL OF THE MECHANICS AND PHYSICS OF SOLIDS
www.elsevier.com/locate/jmps

On shakedown theory for elastic–plastic materials and extensions

Pham Duc Chinh*

Vietnamese Academy of Science and Technology, Vien Co hoc, 264 Doi Can, Hanoi, Viet Nam

Received 24 August 2007; received in revised form 5 November 2007; accepted 13 November 2007

Abstract

The idea that an elastic–plastic structure under given loading history may shake down to some purely elastic state (and hence to a safe state) after a finite amount of initial plastic deformation, can apply to many sophisticated material models with possible allowable changes of additional material characteristics, as has been done in the literature. Despite some claims to the contrary, it is shown; however, that the shakedown theorems in a Melan–Koiter path-independent sense have been extended successfully only for certain elastic–plastic hardening materials of practical significance. Shakedown of kinematic hardening material is determined by the ultimate and initial yield stresses, not the generally plastic deformation history-dependent hardening curve between. The initial yield stress is no longer the convenient one (corresponding to the plastic deformation at the level of 0.2%) as in usual elastic–plastic analysis but to be related to the shakedown safety requirement of the structure and should be as small as the fatigue limit for arbitrary high-cycle loading. Though the ultimate yield strength is well defined in the standard monotonic loading experiment, it also should be reduced to the so-called "high-cycle ratchetting" stress for the path-independent shakedown analysis. A reduced simple form of the shakedown kinematic theorem without time integrals is conjectured for general practical uses. Application of the theorem is illustrated by examples for a hollow cylinder, sphere, and a clamped disk, under variable (including quasiperiodic dynamic) pressure.

Keywords: Plastic collapse; Elastic–plastic material; Path-independent shakedown theorems; High-cycle fatigue; High-cycle ratcheting

1. Introduction

An elastic–plastic structure under given loading history, after a finite amount of initial plastic deformation, and possibly, allowable changes of material characteristics, may eventually shake down to some state, from which it subsequently responds elastically to the external agencies. Otherwise, the structure is considered as having failed, because of continuing plastic deformation or degradation of material properties, beyond allowable limits. Incremental analysis following particular loading histories should be used to study shakedown in the most general sense for all kinds of sophisticated material models, including materials with internal parameters (Zarka and Casier, 1981; König, 1987; Stein et al., 1992;

*Fax: +84 48 333 039.
E-mail address: pdchinh@imech.ac.vn

0022-5096/$ - see front matter © 2007 Elsevier Ltd. All rights reserved.
doi:10.1016/j.jmps.2007.11.005

Corigliano et al., 1995; Weichert and Maier, 2002; Nguyen, 2003; Liu et al., 2005), nonstandard plasticity materials (Pycko and Maier, 1995; Bousshine et al., 2003), heterogeneous material (Magoariec et al., 2004), damaged and cracked materials (Hachemi and Weichert, 1992; Huang and Stein, 1996; Belouchrani and Weichert, 1999), shape-memory material (Feng and Sun, 2006), a gradient plasticity material (Polizzotto, 2008), as well as for the problems with nonlinear geometric effects (Maier, 1972; Weichert and Hachemi, 1998).

The powerful shakedown static and kinematic theorems have been constructed for the elastic perfectly plastic materials by Melan (1938) and Koiter (1963). The essential element of the theorems is their path-independence: The theorems determine the time-independent boundary in the loading space, under which a structure is safe regardless of particular loading histories, while the structure fails if the boundary is violated unrestrictively. With the theorems, one does not need to follow an exact loading history to solve a shakedown problem by the usual direct incremental analysis, and that simplifies the work. Moreover, with any trial admissible static field the static theorem gives a lower bound on the shakedown limit, while with a trial admissible kinematic field the kinematic theorem gives an upper bound. These features (as well as the path-independence) remind us about the respective static and kinematic theorems of plastic limit analysis, which are considered as limiting cases of the shakedown theorems, and the classical minimum energy and complementary energy principles of elasticity.

Many attempts have been made to extend the shakedown theorems to much more sophisticated material models in the literature. However the successes are very limited, as will be critically examined in this paper. We will then give a comprehensive presentation of our path-independent shakedown theorems for kinematic hardening material with illustrating examples of application. The emphasis is given to the determining role of the initial and ultimate yield stresses is shakedown assessment with practical recommendations. A simple reduced form of shakedown kinematic theorem is proposed for applications.

2. Shakedown theorems and extensions

Let $\boldsymbol{\sigma}^{\mathrm{e}}(\mathbf{x}, t)$ denote the fictitious stress response of the body V to external agencies over a period of time ($\mathbf{x} \in V,\ t \in [0, T]$) under the assumption of perfectly elastic behaviour, called a loading process (history). The actions of all kinds of external agencies upon V can be expressed explicitly through $\boldsymbol{\sigma}^{\mathrm{e}}$. At every point $\mathbf{x} \in V$, the elastic stress response $\boldsymbol{\sigma}^{\mathrm{e}}(\mathbf{x}, t)$ is confined to a bounded time-independent domain with prescribed limits in the stress space, called a local loading domain $\mathscr{L}_x$. As a field over V, $\boldsymbol{\sigma}^{\mathrm{e}}(\mathbf{x}, t)$ belongs to the time-independent global loading domain $\mathscr{L}$:

$$\mathscr{L} = \{\boldsymbol{\sigma}^{\mathrm{e}} \mid \boldsymbol{\sigma}^{\mathrm{e}}(\mathbf{x}, t) \in \mathscr{L}_x,\ \mathbf{x} \in V,\ t \in [0, T]\}. \tag{1}$$

In the spirit of classical shakedown analysis, the bounded loading domain $\mathscr{L}$, instead of a particular loading history $\boldsymbol{\sigma}^{\mathrm{e}}(\mathbf{x}, t)$, is given a priori. Shakedown of a body in $\mathscr{L}$ means it shakes down for all possible loading histories $\boldsymbol{\sigma}^{\mathrm{e}}(\mathbf{x}, t) \in \mathscr{L}$. Melan's shakedown static theorem can be stated as (Melan, 1938; Koiter, 1963; Pham, 2003):

$$k_{\mathrm{s}} = \sup_{\boldsymbol{\rho} \in \mathscr{R}} \{k \mid k(\boldsymbol{\rho} + \boldsymbol{\sigma}^{\mathrm{e}}) \in \mathscr{Y},\ \forall \boldsymbol{\sigma}^{\mathrm{e}} \in \mathscr{L}\}, \tag{2}$$

where k_{s} is the shakedown safety factor. At $k_{\mathrm{s}} > 1$ the structure will shake down, while it will not at $k_{\mathrm{s}} < 1$, and $k_{\mathrm{s}} = 1$ defines the boundary of the shakedown domain. $\mathscr{R}$ is the set of (bounded) admissible time-independent self-equilibrated residual stress fields $\boldsymbol{\rho}(\mathbf{x})$ on V; $\mathscr{Y}$ designates the elastic domain in stress space that is bounded by the yield surface (of Mises or Tresca types) determined by the yield stress σ_{Y}.

Let $\mathscr{A}$ denote the set of compatible-end-cycle (deviatoric) plastic strain rate fields $\mathbf{e}^{\mathrm{p}}$ over time cycles $0 \leqslant t \leqslant T$:

$$\mathscr{A} = \left\{\mathbf{e}^{\mathrm{p}} \mid \boldsymbol{\varepsilon}^{\mathrm{p}} = \int_0^T \mathbf{e}^{\mathrm{p}}\, \mathrm{d}t \in \mathscr{C}\right\}, \tag{3}$$