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Multidimensional stability of martensite twins under regular kinetics

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Abstract

This paper considers phase boundaries governed by regular kinetic relations as first proposed by Abeyaratne and Knowles [1990. On the driving traction acting on a surface of strain discontinuity in a continuum. J. Mech. Phys. Solids 38 (3), 345–360; 1991. Kinetic relations and the propagation of phase boundaries in solids. Arch. Ration. Mech. Anal. 114, 119–154]. It shows that static configurations of hyperelastic materials, in which two different martensitic (monoclinic) states meet along a planar interface, are dynamically stable towards fully three-dimensional perturbations. For that purpose, the reduced stability (or reduced Lopatinski) function associated to the static twin [Freistühler and Plaza, 2007. Normal modes and nonlinear stability behavior of dynamic phase boundaries in elastic materials. Arch. Ration. Mech. Anal. 186 (1), 1–24] is computed numerically. The results show that the interface is weakly stable under Maxwellian kinetics expressing conservation of energy across the boundary, whereas it is uniformly stable with respect to linearly dissipative kinetic rules of Abeyaratne and Knowles type.

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Keywords: Martensite twin; Multidimensional stability; Kinetic relations; Lopatinski function; Static phase boundary

1. Introduction

At appropriate temperatures, certain crystalline materials are characterized by a multiplicity of preferred *martensitic states* of deformation. Nonlinear elasticity theory (Ball and James, 1987, 1992) models such materials by a stored-energy density

$$W : \mathbb{R}_+^{3\times 3} \longrightarrow [0, +\infty) \tag{1}$$

which, as a frame-indifferent function of the deformation gradient $\mathbf{F} \in \mathbb{R}_+^{3\times 3}$, has a non-convex, multiple well structure with several global minima. Being any of these minima a martensitic deformation state, each corresponding well is called a *martensitic phase*. Many such materials allow for pairs $(\underline{\mathbf{F}}^-, \underline{\mathbf{F}}^+)$ of energy

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minimizing martensitic states which satisfy the *Hadamard condition*,

$$\underline{\mathbf{F}}^+ - \underline{\mathbf{F}}^- = \mathbf{a} \otimes \boldsymbol{v} \tag{2}$$

for some $\mathbf{a}, \boldsymbol{v} \in \mathbb{R}^3$, $\mathbf{a} \neq 0, |\boldsymbol{v}| = 1$. In this case, $\underline{\mathbf{F}}^-$ and $\underline{\mathbf{F}}^+$ are called *rank-one connected*. Hadamard condition implies the existence of a continuous deformation with planar interface separating layers in which the deformation gradient is either $\underline{\mathbf{F}}^+$ or $\underline{\mathbf{F}}^-$ at each side of the plane. Such a configuration,

$$\mathbf{F}_0(x) = \begin{cases} \underline{\mathbf{F}}^-, & x \cdot \boldsymbol{v} < 0, \\ \underline{\mathbf{F}}^+, & x \cdot \boldsymbol{v} > 0, \end{cases} \quad x \in \mathbb{R}^3, \tag{3}$$

is called a *martensite twin*.

Let us identify the elastic body at rest with its reference configuration $\Omega \subset \mathbb{R}^3$. Mathematically, the function

$$(\mathbf{F}_*, \mathbf{v}_*)(x,t) := (\mathbf{F}_0(x), 0), \quad (x,t) \in \Omega \times [0, +\infty],$$

is a time-independent (static) weak solution to the equations of non-thermal elastodynamics in the absence of external forces,

$$\begin{aligned} &\mathbf{F}_t - \nabla_x \mathbf{v} = 0, \\ &\mathbf{v}_t - \mathrm{div}_x \sigma(\mathbf{F}) = 0, \end{aligned} \tag{4}$$

together with the constraint

$$\mathrm{curl}_x\, \mathbf{F} = 0. \tag{5}$$

Here, $\sigma(\mathbf{F})$ denotes the (first) Piola–Kirchhoff stress tensor

$$\sigma = \frac{\partial W}{\partial \mathbf{F}},$$

and the spatial and temporal derivatives of the local deformation $\mathbf{X} : \Omega \times [0, +\infty) \to \mathbb{R}^3$,

$$\mathbf{F}(x,t) = \nabla_x \mathbf{X} \in \mathbb{R}^{3\times 3} \quad \text{and} \quad \mathbf{v}(x,t) = \mathbf{X}_t \in \mathbb{R}^3 \tag{6}$$

(defined component-wise by,

$$\mathbf{F}_{ij} = \frac{\partial \mathbf{X}_i}{\partial x_j} \quad \text{and} \quad \mathbf{v}_i = \frac{\partial \mathbf{X}_i}{\partial t},$$

for all $i,j = 1,2,3$), denote the possibly time-dependent deformation gradient and local velocity, respectively, of the elastic material.

The system of equations (4) accounts for the basic balance laws of continuum mechanics (see, e.g., Dafermos, 2005), assuming that no thermal effects play a role, and that the forces within the medium derive from the energy density function W. The curl-free constraint (5) is a short-cut for the compatibility equations

$$\partial_{x_k} \mathbf{F}_{ij} - \partial_{x_j} \mathbf{F}_{ik} = 0, \quad i,j,k = 1,2,3,$$

which are clearly a consequence of Eqs. (6).

This paper addresses the stability of martensite twins under the viewpoint of continuum elastodynamics. More precisely, given a multidimensional smooth perturbation (or a small wave impinging on the interface) of the piecewise smooth initial data (3), is there a local solution to Eqs. (4) with the same wave pattern? In other words, will solutions $(\mathbf{F}, \mathbf{v})$ to the dynamic system of equations, whose initial data $(\mathbf{F}, \mathbf{v})(x, 0)$ are only near to—but not identical with—$(\mathbf{F}_0(x), 0)$, be close and similar to—or far and qualitatively different from—$(\mathbf{F}_*, \mathbf{v}_*)$? Considering Eq. (3) as a localized planar sharp interface, we are then concerned with the well-posedness of the associated Cauchy problem for Eqs. (4) with piecewise smooth initial data (3).

Localized traveling discontinuities arise naturally as piecewise smooth solutions to Eqs. (4), and must satisfy canonical jump conditions of Rankine–Hugoniot type expressing conservation across the interface. They are called *shocks* when they separate states in the same martensitic phase, and *phase boundaries* when the states on either side are in different phases. Martensite twins like Eq. (3) are static examples of the latter. Due to non-convexity of the stored energy W, the system of conservation laws is of mixed-type, that is, the system changes