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Accuracy analysis of 3-DOF planar parallel robots

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Abstract

Three-degree-of-freedom planar parallel robots are increasingly being used in applications where precision is of the utmost importance. Clearly, methods for evaluating the accuracy of these robots are therefore needed. The accuracy of well designed, manufactured, and calibrated parallel robots depends mostly on the input errors (sensor and control errors). Dexterity and other similar performance indices have often been used to evaluate indirectly the influence of input errors. However, industry needs a precise knowledge of the maximum orientation and position output errors at a given nominal configuration. An interval analysis method that can be adapted for this purpose has been proposed in the literature, but gives no kinematic insight into the problem of optimal design. In this paper, a simpler method is proposed based on a detailed error analysis of 3-DOF planar parallel robots that brings valuable understanding of the problem of error amplification.

Keywords: Parallel mechanisms; Accuracy; Dexterity; Performance evaluation; Error analysis

1. Introduction

Parallel robots are increasingly being used for precision positioning, and a number of them are used as three-degree-of-freedom (3-DOF) planar alignment stages. Clearly, in such industrial applications, accuracy is of the utmost importance. Therefore, simple and fast methods for computing the accuracy of a given robot design are needed in order to use them in design optimization procedures that look for maximum accuracy.

Errors in the position and orientation of a parallel robot are due to several factors:

– manufacturing errors, which can however be taken into account through calibration;
– backlash, which can be eliminated through proper choice of mechanical components;
– compliance, which can also be eliminated through the use of more rigid structures (though this would increase inertia and decrease operating speed);
– active-joint errors, coming from the finite resolution of the encoders, sensor errors, and control errors.

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Therefore, as pointed out by Merlet [1], active-joint errors (*input errors*) are the most significant source of errors in a properly designed, manufactured, and calibrated parallel robot. In this paper, we address the problem of computing the accuracy of a parallel robot in the presence of active-joint errors only. In the balance of the paper, the term "accuracy" will therefore refer to the position and orientation errors of a parallel robot that is subjected to active-joint errors only.

The classical approach consists in considering the first order approximation that maps the input error to the output error:

$$\delta\mathbf{p} = \mathbf{J}\delta\mathbf{q}, \tag{1}$$

where $\delta\mathbf{q}$ represents the vector of the active-joint (input) errors, $\delta\mathbf{p}$ the vector of output errors and $\mathbf{J}$ is the Jacobian matrix of the robot. However, this method will give only an approximation of the output maximum error. Indeed, as we will prove in this paper, given a nominal configuration and some uncertainty ranges for the active-joint variables, a local maximum position error and a local maximum orientation error not only occur at different sets of active-joint variables in general, but these active-joint variables are not necessarily all at the limits of their uncertainty ranges.

Several performance indices have been developed and used to roughly evaluate the accuracy of serial and parallel robots. A recent study [2] reviewed most of these performance indices and discussed their inconsistencies when applied to parallel robots with translational and rotational degrees of freedom. The most common performance indices used to indirectly optimize the accuracy of parallel robots are the dexterity index [3], the condition numbers, and the global conditioning index [4]. However, in a recent study of the accuracy of a class of 3-DOF planar parallel robots [5], it was demonstrated that dexterity has little to do with robot accuracy, as we define it.

Obviously, the best accuracy measure for an industrial parallel robot would be the maximum position and maximum orientation errors over a given portion of the workspace [1,5] or at a given nominal configuration, given actuator inaccuracies. A general method based on interval analysis for calculating close approximations of the maximum output error over a given portion of the workspace was proposed recently in [1]. Obviously, the maximum output error over a given portion of the workspace is the most important information for a designer. However, this method is relatively difficult to implement, gives no information on the evolution of the accuracy of the manipulator within its workspace and gives no kinematic insight into the problem of optimal design. In contrast, a very simple geometric method for computing the exact value of the accuracy of 3-DOF 3-$\underline{P}RP$ planar parallel robots was described in [5] (in this paper, P and R stand for passive prismatic and revolute joins, respectively, while $\underline{P}$ and $\underline{R}$ stand for actuated prismatic and revolute joins, respectively). This method proposes to replace the existing dexterity maps by maximum position error maps and maximum orientation error maps. While this method covers three of the most promising designs for precision parallel robots (one of which is commercialized and the other two built into laboratory prototypes), it does not always work for other 3-DOF planar parallel robots.

This paper generalizes the method proposed in [5] by following a detailed mathematical proof which gives us important insight into the accuracy of planar parallel robots. The present study considers only 3-DOF three-legged planar parallel robots with prismatic and/or revolute joints, one actuated joint per leg, and at most one passive prismatic joint in a leg. The method is illustrated on two practical designs:

1. A 3-$\underline{R}PR$ planar parallel robot. This robot is the planar projection of the PAMINSA robot [6] and the design parameters are those of the prototype manufactured at INSA of Rennes, France.
2. A planar 3-$\underline{P}RR$ robot [7]. A precision parallel robot based on this design has been developed in the Technical University of Braunscheig, in Germany [8].

The remainder of this paper is organized as follows. Section 2 briefly outlines the mathematical theorems used in this paper. Section 3 presents the method used for the analysis of the orientation and position errors. Finally, Section 4 covers several numerical examples, and conclusions are given in the last section.