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Application of anisotropic inclusion theory to the deformation of Ni based single crystal superalloys: Stress–strain curves determination

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ABSTRACT

The development of plastic deformation after uniaxial plastic strain along [0 0 1] is analyzed in two types of domains in γ–γ′ nickel superalloys. These domains are the horizontal matrix channels normal to [0 0 1] and the vertical channels normal to [1 0 0] and [0 1 0]. By using a mean field method, an elastic energy increase due to the introduction of plastic strain (elongation or compression) in the two types of domains is calculated. The analysis of a mixed mode, where horizontal and vertical channel deformation occurs, is also conducted. Results show that plastic deformation is primarily initiated in a particular type of channel. The choice of the deformation configuration is related to the sign of misfit strain. After a critical amount of plastic strain is reached, the deformation expands through all the matrix channels. This conclusion is supported by the dependence of a flow stress on the deformation mode. FEM calculations are also conducted and compared with the analytical calculation. Except when the strain is extremely small, the two methods give almost the same result.

1. Introduction

Deformation processes in single crystal nickel based superalloys have been investigated by several authors during the past decades. Most results show that a preferred channel deformation occurs when the magnitude of plastic strain is small. The choice of channel deformation is strongly related to the sign of precipitate misfit strain and plastic deformation (elongation or compression) (Pollock and Argon, 1994; Ratel et al., 2009). When the largest component in magnitude of plastic strain is along [0 0 1], horizontal channels are defined as matrix subdomains normal to [0 0 1], and vertical channels are defined as those normal to [1 0 0] and [0 1 0]. There are several crucial experimental observations, obtained by transmission electron microscopy (Feller-Kniepmeier and Link, 1989; Pollock and Argon, 1994), or X-ray diffraction (Khun et al., 1991), which have reported a localized plastic deformation in the horizontal matrix channels during the early stages of a tensile creep test in alloys having a negative lattice parameter mismatch ($a_{\gamma'} < a_{\gamma}$). These results were explained by noting that the superposition of the internal stress due to misfit strain and the applied stress resulted in different forces acting on gliding matrix dislocations. The matrix channels oriented normal to a tensile external stress, which experience a higher total stress field, yield before those oriented parallel to the applied stress, which experience a lower total stress. The roles of channels, normal and parallel to the applied stress, are reversed when misfit strain is positive ($a_{\gamma'} > a_{\gamma}$).

There are important theoretical studies with respect to the channel deformation. FEM analysis was performed (Socrate and Parks, 1993; Pollock and Argon, 1994) in order to evaluate the total stress field in the matrix channels. The results of the calculations conducted by Pollock and Argon indicated that horizontal (vertical) matrix channels experienced a larger von Mises equivalent stress under an

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external tensile stress, when γ' particles had a negative (positive) lattice parameter misfit with respect to the matrix. Accordingly, they showed that the channels which experienced larger stress yielded before those experiencing smaller stress. Socrate and Parks used an energy-momentum tensor, in the context of isotropic elasticity, in order to determine the force acting on a γ–γ' interface and correctly predicted the occurrence of rafting when the amount of plastic strain in the matrix was larger than two times the initial misfit value. Under these conditions, the initial misfit and elastic properties cease to be important (Nabarro et al., 1996). In the previous investigation by (Ratel et al., 2009), the anisotropic inclusion theory combined with a mean field approach has been used to investigate the rafting phenomenon, through evaluation of the elastic energy. In this study, the elastic energy increase associated with the channel deformation was evaluated. It is found that the plastic deformation initiates in a particular type of matrix channels (horizontal or vertical), whose choice depends on the sign of precipitate misfit strain for a given macroscopic strain. The results indirectly show that the plastic deformation tends to become uniform through the matrix when macroscopic strain increases. However, this analysis does not propose any mechanism of the transition from a single type of matrix channel deformation to a quasi uniform matrix deformation. In the present paper, we aim at completing this analysis, by providing the evaluation of the elastic energy increase associated with the operation of both horizontal and vertical matrix channels during plastic deformation. The method employed here is essentially the same as that used in the previous analyses (Ratel et al., 2006; 2009). The calculations are first carried out analytically. In addition, an original approach leading to the determination of stress-strain curves is also presented, for all the mechanisms analyzed in the present investigation. The results obtained in this way are then compared with FEM calculations.

2. Elastic energy evaluation

In order to conduct the analysis in a simple and intuitive way, we assume that the γ matrix and γ' phases have the same elastic constants **C**. This is firstly because our previous paper has shown that this assumption gives nearly the same result as the analysis when γ' is 15% elastically harder than γ. Secondly, because this assumption still predicts the occurrence of rafting in a successful manner without the complication of elastic mismatch (Ratel et al., 2006). In the beginning, the elastic state due to the presence of precipitate misfit strain ε_0 between the γ and γ' phases is determined. Then, the elastic energy change associated with the introduction of plastic strain in the matrix, horizontal and/or vertical channels is calculated. The channel geometry used in the present investigation is described in Fig. 1.

The total elastic energy increase related to a mode of plastic deformation includes a self energy term and an interaction term. Each term is evaluated using the following procedure

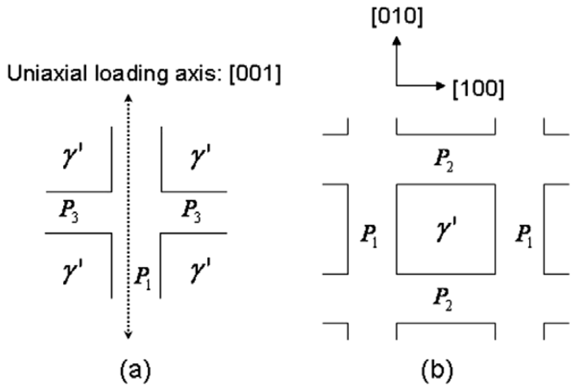

Fig. 1. Geometry of precipitates and different matrix channels configurations. (a) A horizontal channels P_3 and a vertical channel P_1 (the dashed line is the uniaxial loading axis) and (b) two types of vertical channels, P_1 and P_2. The loading direction is normal to the sheet plane.

The stress inside an inclusion is calculated by

$$\boldsymbol{\sigma}^I = \mathbf{C}(\mathbf{S} - \mathbf{I})\varepsilon^*, \tag{1}$$

where **S** is the Eshelby tensor of disk shape inclusions and ε^* the eigenstrain tensor (here the plastic deformation in a matrix channel). The Eshelby tensor of $S_{3333} = 1$ and $S_{3311} = S_{3322} = C_{12}/C_{11}$ is used (Mura, 1987).

The average stress is calculated using the mean field method (Mori and Tanaka, 1973; Brown, 1973).

$$\langle\sigma_{ij}\rangle_V = (1-f)\sigma^I_{ij} \text{ in the particles}, \tag{2}$$

$$\langle\sigma_{ij}\rangle_M = -f\sigma^I_{ij} \text{ in the matrix}. \tag{3}$$

The elastic self energy is calculated using

$$E_P = -\frac{1}{2}\int_D \langle\sigma_{ij}\rangle\varepsilon^*_{ij}dV, \tag{4}$$

where D is the whole body (matrix + particles). The additional energy of the interaction term between two internal stress sources (1) and (2) are given by

$$E_I = -\int_D \langle\sigma_{ij}(1)\rangle\varepsilon^*_{ij}(2)dV. \tag{5}$$

In our case, the sources for internal stress are (1) plastic deformation in a matrix channel and (2) precipitate misfit strain. In the case of the deformation mode where both horizontal and vertical channels operate, an elastic energy of interaction between plastic deformation in horizontal matrix channels and vertical ones must be taken into account. It is similarly calculated, using (5).

In this way, the total elastic energy change induced by the introduction of plastic strain in each material domain considered in the present analysis is evaluated. The deformation mode, leading to the lowest elastic energy increase, is then considered to be the selected one.

2.1. Horizontal channel deformation

Here, we consider that plastic deformation occurs only in the horizontal channels. The plastic strain in the horizontal channels, called P_3 (see Fig. 1), is

$$\varepsilon^P_{11} = \varepsilon^P_{22} = -\varepsilon_P(H)/2, \quad \varepsilon^P_{33} = \varepsilon_P(H). \tag{6}$$