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Effective in-plane elastic properties of auxetic honeycombs with spatial irregularity

T. Mukhopadhyay*, S. Adhikari

College of Engineering, Swansea University, Swansea, UK

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ABSTRACT

An analytical framework has been developed for predicting the equivalent in-plane elastic moduli (longitudinal and transverse Young's modulus, shear modulus, Poisson's ratios) of irregular auxetic honeycombs with spatially random variations in cell angles. Employing a bottom up multi-scale based approach, computationally efficient closed-form expressions have been derived in this article. This study also includes development of a highly generalized finite element code capable of accepting number of cells in two perpendicular directions, random structural geometry and material properties of irregular auxetic honeycomb and thereby obtaining five in-plane elastic moduli of the structure. The elastic moduli obtained for different degree of randomness following the analytical formulae have been compared with the results of direct finite element simulations and they are found to be in good agreement corroborating the validity and accuracy of the proposed approach. The transverse Young's modulus, shear modulus and Poisson's ratio for loading in transverse direction (effecting the auxetic property) have been found to be highly influenced by the structural irregularity in auxetic honeycombs.

1. Introduction

The materials having negative Poisson's ratio are called auxetic material, which exhibits an unusual yet fascinating property of being thicker in dimension perpendicular to the direction of stretching and vice-versa (Evans and Alderson, 2000; Evans et al., 1991). The auxetic behavior in global response of these materials is developed by some specific arrangement of the micro-structural geometry, which allows the material to deform in a particular manner that results the negative Poisson's ratio (refer Fig. 1, for demonstration of auxetic and non-auxetic behaviour of hexagonal honeycombs). This class of materials have attracted considerable attention in last few decades due to their unusual characteristics leading to several application specific desirable engineering properties such as high indentation resistance, increased shear stiffness, increased plane strain fracture toughness, enhanced resistance to buckling under pure bending, superior permeability, enhanced acoustic absorption capacity and direct structural properties like formation of double curvature under flexure (Yang et al., 2004; Evans and Alderson, 2000; Overaker et al., 1998; Stavroulakis, 2005; Critchley et al., 2013; Yao et al., 2008; Bacigalupo and Bellis, 2015; Bacigalupo and Gambarotta, 2014; Scarpa et al., 2003; Yang et al., 2015; Grima et al., 2015; Rad et al., 2014; Nkansah et al., 1994). Moreover natural as well as man made auxetic materials and structural forms can be found across different length-scales strating from nano to macro scale (Evans and Alderson, 2000), wherein the underlying theory of elasticity for analysing mechanical properties of these materials remain same. Thus study of the mechanics behind different forms of such materials have always been of

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* Corresponding author.
E-mail addresses: 800712@swansea.ac.uk, mukhopadhyay.mail@gmail.com (T. Mukhopadhyay), S.Adhikari@swansea.ac.uk (S. Adhikari).
URL: http://www.tmukhopadhyay.com (T. Mukhopadhyay)

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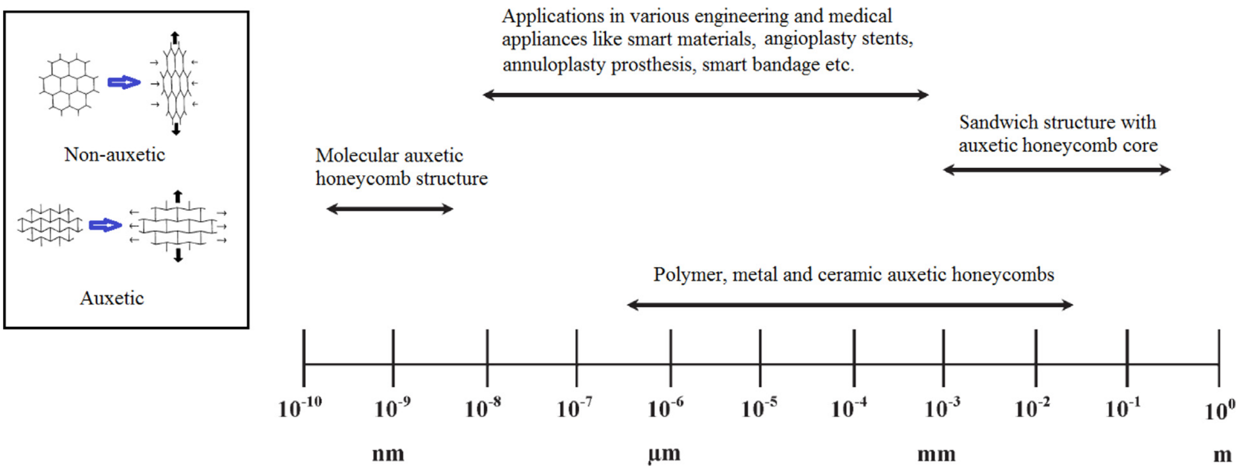

Fig. 1. Occurrence and application of auxetic hexagonal honeycombs across the length scales.

profound interest to the research community since the discovery of auxetic materials. For details about different forms of auxetic materials readers can refer to (Evans and Alderson, 2000). The present article concentrates on auxetic re-entrant hexagonal honeycombs, a brief review of which is presented in the next paragraph.

In last couple of decades, application of hexagonal auxetic lattice forms have been explored from atomic scale to macro scale, in a vast domain ranging from engineering to bio-medical technology. Fig. 1 shows the presence of auxetic re-entrant honeycombs of hexagonal structural form in different length scales. Some of the previous articles (Evans and Alderson, 2000; Nkansah et al., 1994; Karnessis and Burriesci, 2013; Wang and Hu, 2014; Sun et al., 2014; Scarpa and Tomlinson, 2000) have discussed this considering different forms of auxetic materials, in general. Fig. 1 emphasizes on the occurance and application of hexagonal auxetic honeycombs only, according to the aim and scope of the present work. Auxetic character in such hexagonal lattices develops due to the re-entrant shape of their unit cell, as easily perceivable from the figure. The closed form formulae for regular honeycombs provided by Gibson and Ashby (1999) are widely used to obtain in-plane elastic moduli of auxetic honeycombs. Recently new analytical formulae for regular hexagonal honeycombs have been reported considering the nodes at the intersections of inclined and vertical members by Malek and Gibson (2015). Effect of hierarchy in lattices have also been studied by many researchers (Banerjee, 2014). Gereke et al. (2012) have presented a multi-scale stochastic modelling approach for elastic properties of strand-based wood composites. Several studies can be found in available literature dealing with mechanical properties of regular auxetic honeycombs using numerical and experimental investigations (Scarpa et al., 2000; Scarpa and Tomlin, 2000; Grima et al., 2013; Scarpa et al., 2003). Available analytical approaches for obtaining equivalent elastic moduli of auxetic honeycombs are based on unit cell approach, which fails to account for any form of irregularity in the structure. Spatial irregularity in auxetic honeycomb may occur due to uncertainty associated with manufacturing in macro-level and process of fabrication and synthesizing in molecular level, uncertain distribution of intrinsic material properties, structural defects, variation in temperature, pre-stressing and micro-structural variability. Several investigations to explore the effect of irregularity are found to have concentrated on non-auxetic honeycombs including voronoi honeycombs (Li et al., 2007; Triantafyllidis and Schraad, 1998; Ajdari et al., 2008; Zhu et al., 2001; Li et al., 2005; Alsayednoor et al., 2013; Mukhopadhyay and Adhikari, 2016). Papka and Kyriakides (1994) have carried out numerical and experimental study of honeycomb crushing behaviour considering geometrical imperfections in the structure such as variation in length of bond line and over or under expanded cells. The state of available literature which investigates the effect of irregularity in auxetic honeycombs is very scarce. Recently (Liu et al., 2014) have investigated manufacturing irregularity in auxetic honeycomb using finite element analysis. They have reported that structural irregularity influences the effective elastic modulus, yield strength and Poisson's ratio of auxetic honeycombs. All the studies mentioned above exploring the effect of irregularity in both auxetic as well as non-auxetic honecombs are based on either finite element simulations or experimental investigations. Experimental investigations are very expensive and time consuming, which makes it practically not feasible to capture the effect of random irregularities in honeycomb structure in a robust and comprehensive manner by testing huge number of samples. The finite element approach becomes quite computationally intensive because a small change in geometry of a single cell may require creating completely new geometry and meshing the entire structure. Moreover when irregular honeycombs are modelled as a part of another host structure, the degree of freedom to be considered becomes very high making it prohibitively expensive to simulate. The problem becomes even worse for uncertainty quantification using a Monte Carlo based approach (Hurtado and Barbat, 1998; Dey et al., 2015a; 2015b), where the expensive finite element model is needed to be simulated for a large number of samples with different structural configurations. A simple yet computationally efficient way to