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Dynamic void nucleation and growth in solids: A self-consistent statistical theory

T.W. Wright[a,b,*], K.T. Ramesh[b]

[a]US Army Research Laboratory, WMRD, 4600 Deer Creek Loop, Aberdeen Proving Ground, MD 21005, USA
[b]The Johns Hopkins University, Baltimore, MD 21218, USA

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Abstract

We present a framework for a self-consistent theory of spall fracture in ductile materials, based on the dynamics of void nucleation and growth. The constitutive model for the material is divided into elastic and "plastic" parts, where the elastic part represents the volumetric response of a porous elastic material, and the "plastic" part is generated by a collection of representative volume elements (RVEs) of incompressible material. Each RVE is a thick-walled spherical shell, whose average porosity is the same as that of the surrounding porous continuum, thus simulating void interaction through the resulting lowered resistance to further void growth. All voids nucleate and grow according to the appropriate dynamics for a thick-walled sphere made of incompressible material. The macroscopic spherical stress in the material drives the response in all volume elements, which have a distribution of critical stresses for void nucleation, and the statistically weighted sum of the void volumes of all RVEs generates the global porosity. Thus, macroscopic pressure, porosity, and a distribution of growing microscopic voids are fully coupled dynamically. An example is given for a rate-independent, perfectly plastic material. The dynamics of void growth gives rise to a rate effect in the macroscopic material even though the parent material is rate independent.

Keywords: Spall; Void growth; Self-consistent

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1. Introduction

Spall fracture has been intensively studied in many research centers throughout the world for more than four decades; a recent survey of the field was given by Antoun et al. (2003). Data contained therein display spall strengths for a number of materials over many orders of magnitude of volumetric strain rate. Because the data generally seem to follow a straight line in a log–log plot (within experimental error), a formula of the form $\sigma^{\text{spall}} = A(\dot{V}/V_0)^m$ has been used to summarize a large body of information, where A and m are material constants.

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[*] Corresponding author. US Army Research Laboratory, WMRD, 4600 Deer Creek Loop, Aberdeen Proving Ground, MD 21005, USA. Tel.: +1 410 306 1943; fax: +1 410 306 0666.

E-mail address: tww@arl.army.mil (T.W. Wright).

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Because the physics behind the empirical formula seems somewhat obscure, it has seemed worthwhile to re-examine the fundamental physical processes that lead to spall, beginning first with void nucleation and growth. In his thesis Wu (2002) made a comprehensive re-examination of the growth of a single void in a ductile material, which confirmed previous findings of Huang et al. (1991), but also greatly expanded previous work by including inertia, work hardening, rate hardening, thermal softening, heat conduction, and strain gradient effects, see also Wu et al. (2003a–c). For the purposes of the present paper, two main conclusions may be drawn from this body of work: (a) void nucleation in a ductile material may be regarded as a bifurcation in the solution for uniformly applied spherical stress and (b) inertia is a major mechanism that initially inhibits, but later sustains void growth. Furthermore, the critical bifurcation stress under quasi-static loading may be calculated using only standard tensile stress–strain constitutive information.

Many authors have assumed that void growth in a solid will be dominated by viscous effects, e.g., Seaman et al. (1976), Davison et al. (1977), Eftis and Nemes (1991), and Addesio and Johnson (1993). The empirical formula for spall strength would also seem to lend support to this idea. However, Wu's work indicated that although the earliest part of void growth may be influenced by viscous effects, after a very short time local inertia around the growing void becomes the dominating influence. This notion is also supported by the work of Ortiz and Molinari (1992) and Tong and Ravichandran (1995). Therefore, in this paper we focus on inertial effects, leaving inherent material rate effects for later study.

Molinari and Wright (2005) began exploration of the influence of local inertia on void growth once nucleation has occurred. Using the idea that microstructure in the material would cause a statistical distribution of local critical stresses, they explored some of the consequences of local inertia around the expanding voids. Since the evolution of porosity and spherical stress were uncoupled in that work, and because no representation for void interaction was included, only the earliest stages of growth following nucleation could be examined. Nevertheless, it became clear that for a given final porosity achieved at the higher rates of loading, local inertia would lead to the nucleation and growth of many small voids with higher critical stresses, and at lower rates of loading, it would lead to the nucleation and growth only of fewer but larger voids with lower critical stresses. Further, local inertia around the voids gave an apparent rate effect for the spherical stress, because the theory predicted that at a given porosity the stress would be larger for a higher rate of loading than for a lower rate of loading.

Wright et al. (2006) posed the problem of void growth as a dynamic self-consistent system, but with all voids nucleated simultaneously. In their system, a delta function replaced a distribution of critical stresses, and instead of a spherical stress increasing linearly with time, they assumed a constant rate of volumetric expansion so that spherical stress and porosity were fully coupled variables to be found simultaneously. They showed that porosity increased intermittently and that the tensile pressure first rose to a maximum, then dropped to a negative value as it began a decaying oscillation towards zero with continuing expansion. The computed graph of tensile pressure vs. porosity can be regarded as a synthetic constitutive relation at a constant volumetric strain rate, and therefore, the falling tensile pressure with increasing porosity can be expected to lead to localization and spall fracture.

In this paper, we return to a description of critical nucleation stresses through use of a probability distribution function (pdf) as a surrogate for complete knowledge of complex microstructure, but we retain the idea of a prescribed rate of volumetric expansion and equations that are fully coupled with porosity and spherical stress. The assumed pdf can also be used to define and compute collective quantities in the material such as rate of kinetic energy per unit volume around the voids and rate of dissipation associated with void growth. Appendix A gives the details of how this is done.

In the following section, we review dynamic averaging concepts, first introduced by Molinari and Mercier (2001), and recast them into a slightly different form. The usefulness of these concepts is that they permit separation of macroscopic effects in the continuum as a whole, regarded as a porous material, from detailed microscopic effects associated with void nucleation and growth, including energetic consequences. In particular, the local kinetic energy associated with void growth is separated from the kinetic energy of translation in the porous medium.

In Section 3, we remind the reader that in uniaxial straining at the first moment of void nucleation, spherical stress and volumetric rate of work are much larger than deviatoric stress and deviatoric rate of work respectively. Consequently, it is worthwhile to examine void nucleation and growth, fully coupled to spherical