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ScienceDirect
Journal of the Mechanics and Physics of Solids
54 (2006) 2544–2553

JOURNAL OF THE MECHANICS AND PHYSICS OF SOLIDS

www.elsevier.com/locate/jmps

Self-similarity in rock cracking and related complex critical exponents

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Received 19 November 2005; received in revised form 25 March 2006; accepted 30 June 2006

Abstract

The purpose of this paper is to further confirm some interpretations we made previously from materials or rock structure in term of the requirement of the introduction of complex critical exponents, where catastrophic brittle fracture is considered as a kind of second-order phase transition by analogy with percolation phenomenon. We propose here, using acoustic emission measurement data, a more complete experimental validation to support our previous conjecture that, “the higher the grain size and power supply, the longer range the interaction, and therefore the higher the imaginary part of complex critical exponents,” [Moura, A., Lei, X.L, Nishizawa, O., 2005. Prediction scheme for the catastrophic failure of highly loaded brittle materials or rocks. J. Mech. Phys. Solids 53(11), 2435–2455].

Keywords: Acoustics; Asymptotic analysis; Inhomogeneous material; Microfracturing; Statistical mechanics

1. Introduction

Establishing a scheme to forecast large earthquake events or the time-to-failure of material has eluded generations of seismologists and mechanical engineers; the scientific community remains puzzled about this challenge. But as observed many times in the past, limitations encountered in a special branch of physics have been overcome with the emergence of new theory following the unification of former theories. Such fusions can

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doi:10.1016/j.jmps.2006.06.010

only be the product of interdisciplinary research. This is why we state that progress towards earthquake prediction or the prevention of material failure is possible only by enlarging the narrow sphere of materials and rocks studied by mechanical engineers.

Recently, the integration of results from materials science, rock mechanics, acoustic emission, seismology, statistical mechanics, and renormalisation group theory gave rise to the emergence of a criticality theory of materials and rock failure that provides a hopefully new way to tackle this challenge (Lei et al., 2004a; Moura et al., 2005). Self-similarity, criticality, and discrete scale invariance are properties that are already shared by many domains of the natural sciences, are considered together in the present study to make a substantial step in progress towards predicting earthquake events and the time-to-failure of material. The use of acoustic emission measurement data is fundamental to capture the statistical nature of fracture from a thermodynamic point of view. Other pioneering global mechanical parameters based approaches integrating fracture and renormalisation group theory in terms of universal fixed points (Sahimi and Arbabi, 1992) have a weaker power of prediction in seismology.

Industrial material such as rock, intrinsically has numerous defects at the mesoscopic scale. This is different from perfect crystals where global failure occurs from a few or even just one unstable crack related to cleavage for instance. Material global failure stems from random spatially distributed microseparations including nucleation, coalescence, micro-crack advance, fibre rupture, interfacial debonding, etc. The multistep character of progressive damage, reflecting the discrete nature of the fracturing process, makes it possible to compare the diffusion of damage with percolation phenomenon (Stauffer and Sornette, 1998). This analogy was the starting point for extensive numerical simulations applied to lattice models of fracture. These studies predicted the existence of scaling laws in the vicinity of fractures within materials or rocks (Sahimi and Arbabi, 1996).

Statistical self-similarity phenomena are encountered in many domains of natural sciences. Self-similar systems are well-known from many natural fractal forms that possess scale invariance, including coastlines, folds, layering, turbulent flows, leaves, clouds, lungs, bacteria cultures, trees, etc. Physically speaking, the hidden underlying process that governs such systems is understood as self-organisation, although the nature of the process is largely unknown (Sornette, 2000; Wilson, 1979). The most reliable branch of physics to study such irreversible processes is non-linear non-equilibrium statistical thermodynamics, however, the construction of specific statistical models is such a difficult task that a new approach has progressively emerged, the so-called renormalisation group theory, which constitutes the appropriate mathematical tool to describe self-similar phenomena (Brown, 1995).

The starting point of this theory comes from the fact that power law is the only function for an observable to verify a scaling relation given that self-similarity entails scaling. In the vicinity of the critical point associated with second-order phase transition, the free energy exhibits such a law. The exponent is then a so-called critical exponent, and the critical point is assimilated to a fixed point.

An important application of great importance for populations exposed to earthquake risk is explanation of the empirical Gutenberg–Richter law in seismology (Gutenberg and Richter, 1954), as a large earthquake event is postulated to be a critical point (Vere-Jones, 1977; Sornette and Sammis, 1995; Sornette, 2005). Foreshocks are analogous to acoustic emissions (AEs), and it follows, although at a smaller scale, that material failure is also a kind of second-order phase transition if the spatial distribution of the heterogeneities is