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Multi-level homogenization of strength properties of hierarchical-organized matrix–inclusion materials

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A B S T R A C T

The present contribution proposes a homogenization methodology for estimating strength properties of elastoplastic matrix–inclusion materials (possible combination of empty pores, fluid-filled pores, and rigid particles), considering a matrix behavior governed by yield surfaces of second order (e.g., Drucker–Prager, Mises–Schleicher, elliptical surface). The procedure considers yielding of the matrix phase and is based on mean-field methods of continuum micromechanics. The main constituents of the theory are the computation of representative stress measures based of an estimated stress distribution and the assumption of full exploitation of strength within the matrix material. It is found that the resulting effective yield surface is in any case again a function of second order, which allows the extension of the proposed method to multiple application, such as e.g., in case of multi-level material systems. Moreover, a differential homogenization procedure characterized by repeated application of the proposed method is introduced for treating materials with high inclusion fractions. Crucial assumptions and obtained effective yield criteria are validated by means of numerical results obtained from finite-element simulations and with results taken from the literature.

1. Introduction

Many man-made and biological materials are hierarchically structured with particles and pores at smaller length scales governing the behavior at the so-called macro-scale. The comprehension of effects occurring at smaller length scales and their influence on the material behavior at the macro-scale is valuable in terms of efficient engineering usage as well as for possible synthesis of new materials.

An often-cited pioneering contribution to this matter is Gurson (1977) where a micromechanics-based yield criterion for porous materials was derived. By means of a kinematic approach of limit analysis, the effective strength of a hollow sphere consisting of a rigid-plastic Mises material was evaluated. Since then, many contributions refined the so-obtained criterion. One approach for this is based on finite-element simulations of representative volume elements such as in Fritzen et al. (2012a,b) and Khdir et al. (2015) for porous Mises and Green-type materials. Findings in Khdir et al. (2014) show that the size distribution of pores only has a minor effect on macroscopic properties at numerical homogenization. Accordingly, available homogenization methodologies are supposed to be equally appropriate for any size distribution, even though they were developed for one-size pores as virtually all analytical approaches. Alternatively to these FEM-based contributions, Pastor et al. (2013) used numerical limit analysis for examining porous Drucker–Prager, Mises–Schleicher, and Green-type materials, highlighting the overestimation of strength in hydrostatic compression by analytical

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approaches for the first two cases. Still, analytical approaches remain important due to their generality with respect to input parameters and computational efficiency. Mean-field methods initially applied for linear problems (e.g., Mori et al. (1973) and Benveniste (1987)) were more recently adopted for non-linearity. Major progress in terms of non-linear homogenization was made by Ponte Castañeda (1991, 1996, 2002) with variational methods using optimally chosen linear comparison composites. Barthélémy and Dormieux (2004); Dormieux et al. (2006) and Maghous et al. (2009) applied non-linear homogenization methods based on the modified secant method to Drucker–Prager matrix materials with either pores or rigid particles for obtaining effective yield surfaces. Porous Mises–Schleicher materials were treated in Lee and Oung (2000) and Durban et al. (2010). Zhou and Meschke (2014) adopted the method of linear comparison composites introduced by Ponte Castañeda (2002) for homogenization of two-phase materials where both phases (matrix and inclusion) follow Drucker–Prager or elliptical strength criteria. In recent contribution, Shen et al. (2012) and He et al. (2013), approaches to two-step homogenization in case of plasticity were presented, dealing with macro-pores and rigid particles in a porous matrix. Therein, the method introduced in Maghous et al. (2009) was used to model the porous matrix material behavior, considering the macro-pore/particle by applying limit analysis on a spherical single-inclusion model. In Barai and Weng (2011) the problem of agglomeration of fibers was met by two-scale mean-field homogenization. Also Shen et al. (2013) applies mean-field methods for two homogenization steps, highlighting the effect of the flow rule (associated and non-associated).

The present contribution focuses on the development of an easily applicable method for multi-phase and multi-level homogenization based on the following assumptions:

- The inclusion phases consist of spherical particles or cavities (empty or fluid-filled) on different length scales.
- The matrix material behavior is representable by perfect plasticity (no hardening or rate effects).
- The material behavior of particles is linear elastic with the stiffness being significantly higher than of the matrix material (quasi-rigid). The stiffness of fluid filled pores is high in compression as well (quasi-incompressible), while negligible in shear.
- The material is subjected to monotonic loading and infinitesimal deformation.

The approach represents an extension of the modified secant method for porous materials (Dormieux et al., 2006), reducing strength homogenization to an elastic substitute problem, with the appropriate choice of elastic parameters enabling the proper estimation of the ultimate limit stress state within the material system. It is applied to matrix–inclusion materials, where the considered matrix material is governed by arbitrary second-order yield surfaces (including, e.g., Mises–Schleicher, Drucker–Prager and elliptical surfaces as special cases):

$$f(\boldsymbol{\sigma}) = \sigma_d^2 + a\sigma_m + b\sigma_m^2 - c \leqslant 0, \tag{1}$$

where σ_m is the hydrostatic pressure, and σ_d the equivalent deviatoric stress:

$$\sigma_m = \frac{1}{3}\mathrm{tr}(\boldsymbol{\sigma}), \quad \sigma_d = \sqrt{\frac{1}{2}\mathbf{s}:\mathbf{s}}, \quad \mathbf{s} = \boldsymbol{\sigma} - \mathbf{1}\sigma_m. \tag{2}$$

The factor $\sqrt{1/2}$ in the definition of σ_d in Eq. (2) ensures that for pure shear $\sigma_d = \tau$ (with τ as the only non-zero component of the stress tensor). Hence, σ_d can also be denoted as the equivalent pure shear. The variables a, b, and c in Eq. (1) denote material parameters of the matrix material; c corresponds to the square of the cohesion (i.e., $c = \tau_y^2$, where τ_y is the absolute value of the yield stress at pure shear), and a and b refer to the pressure sensitivity of the matrix material ($a = b = 0$ for Mises matrix materials). Differences between strength in tension and compression can be considered with $a \neq 0$. The Mises–Schleicher criterion is retrieved with $a > 0$ and $b = 0$, and the Drucker–Prager criterion with $a = a_{DP}\tau_y$ and $b = -\tau_y^2$ where a_{DP} is the friction coefficient in the Drucker–Prager yield criterion ($f_{DP}(\boldsymbol{\sigma}) = \sigma_d + a_{DP}\sigma_m - \tau_y = 0$).[2] Since, as later shown, the yield surfaces resulting from the presented homogenization scheme are of second-order as well, a straight forward application to multi-level homogenization becomes possible. Within this framework, a differential homogenization scheme is proposed as an alternative approach for strength homogenization when dealing with materials exhibiting high inclusion fractions. Analytical considerations are accompanied with numerical analyses of a representative elementary volume (REV) using the finite-element method, providing the basis for validation of basic assumptions considered in the derivation and of the obtained effective yield surfaces.

The paper is structured as follows: In Section 2, the homogenization procedure for matrix–inclusion materials is presented exploiting the Mori–Tanaka scheme for obtaining an estimate of the stress state in the material system. Basic assumptions and the model performance are tested via numerical simulations. In Section 3, the extension of the model towards multiple application is presented, leading to multi-level homogenization and the differential scheme of strength homogenization. Examples of two-level homogenization illustrate the influence of material composition and hierarchical organization of materials on the effective strength. The paper closes with concluding remarks. In Appendix A, the main principles and equations of the Mori–Tanaka scheme (Mori et al., 1973; Benveniste, 1987) required in this paper are given. Appendix B contains a part of the derivation of the homogenization scheme presented in Section 2 (i.e., the evaluation of the virtual work in consequence of an eigenstress) in more detail.

2. Model and model validation

For determining the effective behavior of matrix–inclusion materials, a representative elementary volume (REV,

[2] Note, that $f_{DP}(\boldsymbol{\sigma}) = 0$ yields the same yield surface as $f(\boldsymbol{\sigma}) = 0$ in Eq. (1).