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Hertzian fracture at unloading

Denis Elaguine[a], Marie-Anne Brudieu[b], Bertil Storåkers[a,*]

[a]*Department of Solid Mechanics, Royal Institute of Technology, S-100 44 Stockholm, Sweden*
[b]*Ecole Polytechnique, F-91128 Palaiseau, France*

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Abstract

Hertzian fracture through indentation of flat float glass specimens by steel balls has been examined experimentally. Initiation of cone cracks has been observed and failure loads together with contact and fracture radii determined at monotonically increasing load but also during unloading phases. Contact of dissimilar elastic solids under decreasing load may cause crack inception triggered by finite interface friction and accordingly the coefficient of friction was determined by two different methods. In order to make relevant predictions of experimental findings, a robust computational procedure has been developed to determine global and local field values in particular at unloading at finite friction. It was found that at continued loading it is possible to specify in advance how the contact domain divides into invariant regions of stick and slip. The maximum tensile stress was found to occur at the free surface just outside the contact contour, the relative distance depending on the different elastic compliance properties and the coefficient of friction. In contrast, at unloading invariance properties are lost and stick/slip regions proved to be severely history dependant and in particular with an opposed frictional shear stress at the contact boundary region. This causes an increase of the maximum tensile stress at the contour under progressive unloading. Predictions of loads to cause crack initiation during full cycles were made based on a critical stress fracture criterion and proved to be favourable as compared to the experimental results.

Keywords: Contact mechanics; Elastic material; Friction; Fracture; Unloading

*Corresponding author. Tel.: +46 8 790 8641; fax: +46 8 411 2418.
E-mail address: bertil@hallf.kth.se (B. Storåkers).

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1. Introduction

Pressing a hard spherical indenter normally into a brittle material will usually result in a ring crack on the surface which then propagates into a frustum of a cone. This was initially investigated by Hertz (1896) and subsequently many efforts have been made to achieve a satisfactory qualitative and quantitative interpretation describing the initiation and growth of such types of cracks. A detailed account of work done in the field has recently been given by Lawn (1998). There are several reasons to study Hertzian fracture properties. First there is a need to provide a theoretical basis for the Hertzian fracture test—a simple way to examine the strength of brittle materials, cf. e.g. Frank and Lawn (1967), Wilshaw (1971) and Warren (1995). A more practical reason is the need to model indentation damage, which is now recognized to be a key limiting factor in the life-time of many engineering applications, cf. e.g. Flocker and Dharani (1997).

In spite of the existence of a vast literature on the Hertzian indentation system and on the conditions for formation of cone cracks, there is one experimental issue which has up to now received relatively little attention. This is Hertzian fracture during unloading. It has been observed by several investigators that if a brittle specimen has not fractured during indentation to maximum load it might still fail during unloading (Argon et al., 1960; Wilshaw, 1971; Johnson et al., 1973; Geandier et al., 2003). It is the present intention to investigate this somewhat unexpected but important phenomenon from a tangible analytical and experimental background.

A primary attempt to study the phenomenon experimentally dates back to Argon et al. (1960). These writers performed a series of Hertzian fracture tests on polished crown glass with steel balls during a complete load cycle. First the load was monotonically increased to its maximum value, kept at this level, or slightly decreased, for a certain time followed by unloading at the same rate as in the initial phases. It was found, that if a specimen survived during loading, it frequently failed either while the load was maintained at a constant level or during unloading.

Crack initiation in a specimen subjected to a constant load for a certain time has been attributed by Argon et al. (1960) to a time-dependant chemical process on the surfaces of precursor microcracks, as originally explained by Orowan (1944). At the same time, Argon et al. (1960) argued that since numerous fractures occur during load removal in specimens which survived a significant time under maximum load, unloading fracture should be due to a different physical mechanism. It was suggested that crack initiation during unloading is the result of 'a genuine fracturing effect of the load removal', but the mechanism of this process was not clarified in detail.

Later Johnson et al. (1973) carried out a thorough theoretical and experimental investigation dealing with the influence of interfacial friction on Hertzian fracture initiation. It was shown that during monotonically increasing load, friction induces outward shear tractions in the surface of a more compliant body. As a result, the maximum tensile stress will be reduced and shifted from the contact contour. At unloading shear tractions of opposite sign arise at the edge of the contact area. Accordingly the frictional effect which is protective during loading gives rise to a damaging peak tension during unloading.

In order to give an experimental background to the theoretical predictions, Johnson et al. (1973) performed comparative tests on float soda lime silica glass with steel and glass